Formulas that you might want to use.

1. If A is an absorbing stochastic matrix with

$$A = \left[\begin{array}{c|c} I & S \\ \hline 0 & R \end{array}\right]$$

then the stable matrix of A is

$$\left[\begin{array}{c|c} I & S(I-R)^{-1} \\ \hline 0 & 0 \end{array}\right]$$

where the identity matrix I in $(I-R)^{-1}$ is chosen to be the same size as R.

2. Compound Interest.

Compound amount $F = (1 + i)^n P$

Present value $P = \dfrac{F}{(1 + i)^n}$

3. Simple interest.

Amount $A = (1 + nr)P$

MATH 104 - EXAM III

1. Which of the following describes the equation of the line which passes through the point (1, 2) and is parallel to the line, $2y + x = 1$.

(a) $y = -2x + 3$ (b) $y = 2x-2$ (c) $y = 2x$ (d) $y = \frac{-1}{2}x + \frac{1}{2}$ (e) $y = \frac{-1}{2}x + \frac{5}{2}$

2. The price of a gallon of gas at the "Gas'n go" station was \$1.00 on January 1, 1993 and \$1.10 on January 1, 1995. If the price varies linearly with time, what will a gallon of gas cost at "Gas'n go" on January 1, 2001?

(a) \$1.20 (b) \$1.35 (c) \$1.30 (d) \$1.40 (e) \$1.25

3. Find the y intercept of the line that passes through (1,1) and has slope $\frac{1}{2}$.

(a) $y = \frac{-1}{2}$ (b) $y = 0$ (c) $y = -1$ (d) $y = 1$ (e) $y = \frac{1}{2}$

4. Which of the following statements is true about the solution to the following system of equations?

$$x + 2y = -1$$
$$x - y = 2$$

(a) The value of x is -7 (b) The value of x is -1 (c) The value of x is 1
(d) There are infinitely many solutions (e) The system has nosolution

5. The matrix $\begin{bmatrix} 1 & 1 & 3 \\ \textcircled{2} & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ is pivoted around the circled entry. What is the entry in the first row and second column of the resulting matrix?

(a) 3 (b) 0 (c) 4 (d) 1 (e) 2

6. Which of the following statements is true about the solution to the following systems of equations?

$$\begin{cases} x + y + z = 2 \\ 2x + 3y + 5z = 5 \\ y - 2z = 1 \end{cases}$$

(a) There is no solution (b) There are infinitely many solutions
(c) The value of x is 1 (d) The value of x is 0 (e) The value of x is -1

7. Given that $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$, solve for y in the following system of linear equations

$$x + z = 3$$
$$x + y = 1$$
$$x + y + z = 5$$

(a) y = -1 (b) y = any number (c) y = -4
(d) y = 2 (e) There is no solution

8. A system of linear equations has the augmented matrix

$$\begin{array}{cccc|c} x & y & z & w & \\ \hline 1 & 2 & 0 & 0 & 5 \\ 0 & 0 & 1 & 2 & 4 \end{array}$$

What is the general solution to the system?

(a) y = any number
w = any number
x = 5
z = 4

(b) y = any number
w = 1
z = 2
x = 5 - 2y

(c) y = any number
w = any number
z = 4 - 2w
x = 5 - 2y

(d) w = any number
z = 4 - 2w
y = 1
x = 3

(e) y = 1, z = 2
w = 1, x = 3

9. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$. Find the entry in the 2nd row and first column of A^{-1}.

(a) $\frac{2}{5}$ (b) -2 (c) $\frac{-3}{5}$ (d) $\frac{3}{5}$ (e) -3

10. Use the Gauss Jordon method to find the entry in the third row and second column of A^{-1} if

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(a) -1 (b) $\frac{-1}{2}$ (c) 0 (d) $\frac{1}{2}$ (e) 1

11. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

Find A + 2B

(a) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \\ 3 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 2 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 2 & 5 \end{bmatrix}$ (e) $\begin{bmatrix} 4 & 3 \\ 1 & 1 \\ 2 & 4 \end{bmatrix}$

12. Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

What is A. B.

(a) $\begin{bmatrix} 3 & 5 & 4 \\ 4 & 5 & 7 \\ 2 & 3 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & 8 \\ 5 & 6 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 4 & 2 \\ 3 & 3 & 6 \\ 1 & 1 & 2 \end{bmatrix}$ (e) not defined

13. Consider the matrices

(I) $\begin{bmatrix} \frac{1}{4} & \frac{7}{3} \\ \frac{3}{4} & \frac{-4}{3} \end{bmatrix}$ (II) $\begin{bmatrix} .6 & 1 \\ .4 & 0 \end{bmatrix}$ (III) $\begin{bmatrix} .4 & .8 \\ .2 & .2 \end{bmatrix}$

(IV) $\begin{bmatrix} .3 & 0 & 1 \\ .7 & .1 & 0 \\ 0 & .9 & 0 \end{bmatrix}$ (V) $\begin{bmatrix} .2 & .4 \\ .1 & .4 \\ .7 & .2 \end{bmatrix}$

Which of them are stochastic matrices

(a) all except V (b) II and IV only (c) all except I
(d) II, III and IV only (e) none of the above

14. Determine the values of x and y such that the following matrix is a transition matrix of stochastic process

$$\begin{bmatrix} .3 & y \\ x & 0.65 \end{bmatrix}$$

(a) x = 1.3, y = .65 (b) x = -0.3, y = 0.0 (c) x = .7, y = .35
(d) x = 0.35, y = .7 (e) x = -0.65, y = -0.3

15. Suppose that the people in a certain city are catching cold. It is observed that after one week, 40% of the people who were sick are still sick and of those who were well, 30% are sick. Suppose this trend continues and that on one particular week 20% of the population are sick. Find the proportion of people who are sick after two weeks.

(a) .332 (b) .104 (c) .320 (d) .668 (e) one of the above

16. Which of the following are regular stochastic matrices.

(I) $\begin{bmatrix} .4 & .2 \\ .6 & .5 \end{bmatrix}$ (II) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (III) $\begin{bmatrix} .2 & 0 & 0 \\ .1 & .4 & 0 \\ .7 & .6 & 1 \end{bmatrix}$ (IV) $\begin{bmatrix} .1 & .7 \\ .9 & .3 \end{bmatrix}$ (V) $\begin{bmatrix} 0 & .7 \\ 1 & .3 \end{bmatrix}$

(a) IV and V only (b) IV only (c) I and IV only (d) II and III only
(e) none of the above

17. The stable distribution for the regular stochastic matrix $\begin{bmatrix} .4 & .2 \\ .6 & .8 \end{bmatrix}$ is

(a) $\begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$

(e) none of the above

18. The stable matrix for the absorbing matrix $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{8} \\ 0 & 0 & \frac{3}{8} \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 & \frac{4}{5} \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & \frac{3}{8} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{8} \\ 0 & 0 & \frac{3}{8} \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & \frac{4}{5} \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{8} \\ 0 & 0 & 0 \end{bmatrix}$ (e) none of the above

19. What is the compound amount after 2 years of $100 deposited at 6% interest compounded annually?

(a) $120.00 (b) $112.36 (c) $110.00 (d) $121.00
(e) none of the above

20. How many months are required for $2500.00 to grow to $3300 at 16% simple interest?

(a) 20 (b) 18 (c) 24 (d) 22
(e) none of the above