## MATH 104 - EXAM I

1. Let $U=\{a, b, c, d, e, f, g, h, i, j\}$.

Let $R=\{a, c, e, g, i\}, S=\{b, c, d, e, f, g\}$ and $T=\{a, c, d, f, h, i\}$.
What is $\left(R \cap T^{\prime}\right) \cap S$ ?
(a) $\{c\}$
(b) $\varnothing$
(c) $\{e, g\}$
(d) $\{d, f\}$
(e) $\{a, c, e, f\}$
2. Identify the shaded region in the following diagram:

(a) $(\mathrm{A} \cup \mathrm{B})^{\prime} \cap \mathrm{C}$
(b) $A \cap B \cap C$
(c) $(A \cap B) \cap B^{\prime}$
(d) $(A \cup B) \cap C$
(e) $\mathrm{C} \cup(\mathrm{A} \cap \mathrm{B})$
3. If $R$ and $S$ are finite subsets of a universal set $U$, such that $n\left(R^{\prime}\right)=20, n(S)=15$, $n\left(S \cup R^{\prime}\right)=30$ and $n(U)=35$, how many elements in $S \cap R$ ?
(a) 0
(b) 5
(c) 10
(d) 15
(e) 20
4. A list of the food preferences of 50 species of birds are included with a bird feeder: 30 species like sunflower seed, 20 like millet and 10 like thistle seed. 10 like both sunflower seed and millet, 5 like both sunflower seed and thistle seed and 4 like both millet and thistle seed. Also 2 species like all three of the above types of food. How many like none of the above types of food?
(a) 7
(b) 5
(c) 10
(d) 15
(e) 0
5. At a meeting of the United Nations there are two delegates form each of 150 countries. A committee consisting of 5 people is to be formed where no two delegates are from the same country. In how many ways can this committee be chosen?
(a) $P(150,5) \times 2^{5}$
(b) $C(150,5) \times 2^{5}$
(c) $\mathrm{C}(300,5)$
(d) $\mathrm{P}(300,5)$
(e) $(150)^{5}$
6. In how many ways can you line up 4 boys and 4 girls so that neither two boys nor two girls are standing next to each other?
(a) $P(8,4) \times 2^{4}$
(b) $4!\times 4$ !
(c) $C(8,4) \times 2^{4}$
(d) $2 \times 4!\times 4$ !
(e) 8 !
7. If a set $X$ has 6 elements, how many subsets does it have with 2 or more elements?
(a) $2^{6}-\mathrm{C}(6,0)-\mathrm{C}(6,1)$
(b) $2^{6}$
(c) $\mathrm{C}(6,2)$
(d) $6!-C(6,0)-C(6,1)$
(e) $\mathrm{P}(6,2)$
8. What is the coefficient of $x^{3} y^{2}$ in the expansion of $(x+y)^{5}$ ?
(a) 1
(b) 5
(c) 10
(d) 60
(e) 20
9. An urn contains 3 red balls and 4 green balls. A sample (without replacement) of 3 balls is selected from the urn. How many such samples with 2 red balls and 1 green ball are possible?
(a) $\mathrm{C}(3,2)$
(b) $\mathrm{C}(3,2) \times \mathrm{C}(4,1)$
(c) $\mathrm{C}(4,1)$
(d) $\mathrm{C}(7,3)-4$
(e) $\frac{\mathrm{C}(7,3)}{2!}$
10. The following is a street map of Venice. Antonio lives at A, Portia lives at B and Shylock lives at C. In how many ways can Antonio get from $A$ to $B$ without going past Shylock's house at $C$ if he travels east or south only ?

(a) $\mathrm{C}(9,4)-\mathrm{C}(5,2)(\mathrm{b}) \quad \mathrm{C}(9,4)-1$
(c) $C(5,2) \times C(4,2)$
(d) $\mathrm{C}(9,4)$
(e) $\mathrm{C}(9,4)-\mathrm{C}(5,2) \times \mathrm{C}(4,2)$
11. The Verona Ice-Cream Shop offers a choice of 10 flavors of ice-cream and 5 toppings. Romeo and Juliet decide to share an ice-cream. In how many ways can they select a cone with one flavor of ice-cream and two toppings?
(a) $10 \times P(5,2)$
(b) $10 \times \mathrm{C}(5,2)$
(c) $10+\mathrm{C}(5,2)$
(d) $10+P(5,2)$
(e) $10 \times 5$
12. An experiment consists of flipping a coin 10 times and observing the sequence of heads and tails. How many of the possible sequences contain either exactly 2 heads or exactly 3 heads?
(a) $\mathrm{C}(10,2)+\mathrm{C}(10,3)$
(b) $\mathrm{C}(10,2) \times \mathrm{C}(10,3)$
(c) $2^{2}+2^{3}$
(d) $2^{5}$
(e) $2^{10}-\mathrm{C}(10,1)$
13. The table below relates to crime statistics. The table shows the location of crime and the type of crime.

|  | Robbery | Murder | Assault |
| :--- | :---: | :---: | :---: |
| Residential | 130 | 40 | 30 |
| Commercial | 102 | 90 | 20 |

The probability that a randomly selected crime was committed in a commercial area given that it was an assault is:
(a) $\frac{2}{15}$
(b) $\frac{1}{7}$
(c) $\frac{1}{3}$
(d) $\frac{2}{5}$
(e) none of the above
14. Events $E$ and $F$ are independent and $\operatorname{Pr}(F) \neq 0$. Which of the following must be true?
(a) $\operatorname{Pr}(E \mid F)=0$
(b) $\operatorname{Pr}(E \cap F)=0$
(c) $\operatorname{Pr}(E \mid F)=\operatorname{Pr}(F)$
(d) $\operatorname{Pr}(E \cup F)=\operatorname{Pr}(E)+\operatorname{Pr}(F)$
(e) none of the above
15. A coin is to be tossed 10 times. What is the probability of 7 heads and 3 tails?
(a) $\frac{3}{10}$
(b) $\frac{7}{10}$
(c) $\frac{15}{128}=\frac{C(10,7)}{2^{10}}$
(d) $\frac{9}{128}$
(e) none of the above
16. Let $E$ and $F$ be events such that $\operatorname{Pr}\left(E^{\prime}\right)=0.4, \operatorname{Pr}(F)=0.3$ and $\operatorname{Pr}(E \cup F)=0.7$. What is $\operatorname{Pr}(E \cap F)$ ?
(a) 0.35
(b) 0.25
(c) 0.12
(d) 0.2
(d) none of the above
17. Peter is studying mathematics and politics. The probability that he passes mathematics is 0.75 , the probability that he fails politics is 0.2 and the probability that he passes mathematics but fails politics is 0.05 . The probability that he passes both courses is;
(a) 0
(b) 0.7
(c) 0.6
(d) 0.85
(e) none of the above
18. The letters of the word "reality" are scrambled and arranged in a random order. What is the probability that the resulting arrangement reads "reality"?
(a) $\frac{3}{7}$
(b) $\frac{4}{7}$
(c) $\frac{C(7,2)}{7!}$
(d) $\frac{1}{7!}$
(e) none of the above
19. The probability of John winning the case brought against him by Paula is 0.55 . What are the odds in favor of John winning?
(a) 45 to 55 (b) 45 to 100
(c) 55 to 100
(d) 55 to 45
(e) none of the above
20. Which of the following is a valid probability distribution for a sample space $\{a, b, c\}$ ?
(a) $\operatorname{Pr}(\mathrm{a})=1.0, \quad \operatorname{Pr}(\mathrm{~b})=0.5, \quad \operatorname{Pr}(\mathrm{c})=0.5$
(b) $\operatorname{Pr}(\mathrm{a})=0.7, \quad \operatorname{Pr}(\mathrm{~b})=0.0, \quad \operatorname{Pr}(\mathrm{c})=0.3$
(c) $\operatorname{Pr}(\mathrm{a})=1.4, \quad \operatorname{Pr}(\mathrm{~b})=-0.4, \quad \operatorname{Pr}(\mathrm{c})=0$
(d) $\operatorname{Pr}(\mathrm{a})=0.1, \quad \operatorname{Pr}(\mathrm{~b})=0.2, \quad \operatorname{Pr}(\mathrm{c})=0.1$
(e) none of the above.

