Formulas that you might want to use.

1. If A is an absorbing stochastic matrix with

$$A = \left[\begin{array}{c|c} I & S \\ \hline 0 & R \end{array}\right]$$

then the stable matrix of A is

$$\left[\begin{array}{c|c} I & S(I-R)^{-1} \\ \hline 0 & 0 \end{array}\right]$$

where the identity matrix I in $(I-R)^{-1}$ is chosen to be the same size as R.

2. Compound Interest.

Compound amount $F = (1 + i)^n P$

Present value $P = \dfrac{F}{(1 + i)^n}$

3. Simple interest.

Amount $A = (1 + nr)P$

MATH 104 - EXAM III

1. Which of the following describes the equation of the line which passes through the point (1, 2) and is perpendicular to the line, $2y + x = 1$.

(a) $y = 2x-2$ (b) $y = 2x$ (c) $y = \frac{-1}{2}x + \frac{5}{2}$ (d) $y = \frac{-1}{2}x + \frac{1}{2}$ (e) $y = -2x + 3$

2. The price of a gallon of gas at the "Gas'n go" station was $1.00 on January 1, 1993 and $1.10 on January 1, 1995. If the price varies linearly with time, what will a gallon of gas cost at "Gas'n go" on January 1, 2000?

(a) $1.30 (b) $1.25 (c) $1.35 (d) $1.40 (e) $1.20

3. Find the x intercept of the line that passes through (1,1) and has slope $\frac{1}{2}$.

(a) $x = -1$ (b) $x = \frac{-1}{2}$ (c) $x = 0$ (d) $x = 1$ (e) $x = \frac{1}{2}$

4. Which of the following statements is true about the solution to the following system of equations?

$$x + 2y = -1$$
$$x - y = 2$$

(a) The value of x is -7 (b) The value of x is -1 (c) The system has no solution
(d) There are infinitely many solutions (e) The value of x is 1

5. The matrix $\begin{bmatrix} 1 & 1 & 3 \\ \textcircled{2} & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ is pivoted around the circled entry. What is the entry in the first row and third column of the resulting matrix?

(a) 0 (b) 3 (c) 2 (d) 1 (e) 4

6. Which of the following statements is true about the solution to the following systems of equations?

$$\begin{cases} x + y = 2 \\ 2x - y = 1 \\ 3x + 2y = 4 \end{cases}$$

(a) There is no solution (b) There are infinitely many solutions
(c) The value of x is 1 (d) The value of x is 0 (e) The value of x is $\frac{7}{10}$

7. Given that $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{bmatrix}$, solve for y in the following system of linear equations

$$x + z = 5$$
$$x + y = 3$$
$$x + y + z = 1$$

(a) y = 7 (b) y = any number (c) There is no solution
(d) y = -4 (e) y = -2

8. A system of linear equations has the augmented matrix

$$\begin{array}{cccc|c} x & y & z & w & \\ \hline 1 & 2 & 0 & 0 & 5 \\ 0 & 0 & 1 & 2 & 4 \end{array}$$

What is the general solution to the system?

(a) y = any number
w = any number
x = 5
z = 4

(b) w = any number
z = 4 - 2w
y = 1
x = 3

(c) y = 1, z = 2
w = 1, x = 3

(d) y = any number
w = any number
z = 4 - 2w
x = 5 - 2y

(e) y = any number
w = 1
z = 2

9. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$. Find the entry in the 2nd row and first column of A^{-1}.

(a) $\frac{-3}{5}$ (b) -3 (c) $\frac{3}{5}$ (d) $\frac{2}{5}$ (e) -2

10. Use the Gauss Jordon method to find the entry in the third row and second column of A^{-1} if

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) 0 (d) 1 (e) -1

11. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \\ 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

Find A + 2B

(a) $\begin{bmatrix} 4 & 3 \\ 1 & 1 \\ 2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 2 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 4 & 2 \\ 2 & 1 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 2 & 5 \end{bmatrix}$ (e) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \\ 3 & 5 \end{bmatrix}$

12. Find the entry in the second row and third column of the product,

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

(a) 3 (b) 9 (c) 8 (d) 1 (e) 1.2

13. Consider the matrices

(I) $\begin{bmatrix} \frac{1}{4} & \frac{7}{3} \\ \frac{3}{4} & \frac{-4}{3} \end{bmatrix}$ (II) $\begin{bmatrix} .6 & 1 \\ .4 & 0 \end{bmatrix}$ (III) $\begin{bmatrix} .4 & .8 \\ .2 & .2 \end{bmatrix}$

(IV) $\begin{bmatrix} .3 & 0 & 1 \\ .7 & .1 & 0 \\ 0 & .9 & 0 \end{bmatrix}$ (V) $\begin{bmatrix} .2 & .4 \\ .1 & .4 \\ .7 & .2 \end{bmatrix}$

Which of them are stochastic matrices

(a) all except I (b) II, III and IV only (c) II and IV only
(d) all except V (e) none of the above

14. Determine the values of x and y such that the following matrix is a transition matrix of stochastic process

$$\begin{bmatrix} .3 & y \\ x & 0.65 \end{bmatrix}$$

(a) x = -0.3, y = 0.0 (b) x = 0.35, y = .7 (c) x = 1.3, y = .65

(d) $x = .7, y = .35$ (e) $x = -0.65, y = -0.3$

15. Assume 50% of women currently work. Of those who work, 70% of their daughters will work. Of those who don't work, only 40% of their daughters will work. Find the percentage of women in the next generation who work.

(a) 70% (b) 55% (c) 50% (d) 40% (e) none of the above

16. Consider regular matrix $\begin{bmatrix} .3 & .4 \\ .7 & .6 \end{bmatrix}$. In order to find its stable distribution, it is necessary to solve the system of equations.

(a) $\begin{cases} x + y = 1 \\ .3x + .4y = x \\ .7x + .6y = y \end{cases}$ (b) $\begin{cases} x + y = 1 \\ .3x + .4y = 0 \\ .7x + .6y = 0 \end{cases}$

(c) $\begin{cases} x + y = 0 \\ .3x + .4y = 1 \\ .7x + .6y = 1 \end{cases}$ (d) $\begin{cases} x + y = 1 \\ .3x + .7y = x \\ .4x + .6y = y \end{cases}$

(e) none of the above

17. The stable distribution for the regular stochastic matrix $\begin{bmatrix} .4 & .2 \\ .6 & .8 \end{bmatrix}$ is

(a) $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{3}{4} & \frac{3}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

(e) none of the above

18. The stable matrix for the absorbing matrix $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{8} \\ 0 & 0 & \frac{3}{8} \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{8} \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{8} \\ 0 & 0 & \frac{3}{8} \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & \frac{4}{5} \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & \frac{3}{8} \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & \frac{1}{5} \\ 0 & 1 & \frac{4}{5} \\ 0 & 0 & 0 \end{bmatrix}$ (e) none of the above

19. What is the compound amount after 2 years of $100 deposited at 10% interest compounded annually?

(a) $121.00 (b) $110.00 (c) $120.00 (d) $121.55
(e) none of the above

20. How many months are required for $2500.00 to grow to $3300 at 16% simple interest?

(a) 18 (b) 20 (c) 22 (d) 24
(e) none of the above