1. Let **U** = {a, b, c, d, e, f, g, h, i, j } and let
R = {a, c, e, g, i } ; **S** = {b, c, d, e, f } ; **T** = {a, b, f, g, h}

Which of the sets below is **(R' ∩ S) ∪ T** ?

(a) {c, e, f} (b) {b, d, f } (c) {a, b, d, f, g, h } (d) {e, g }

(e) ∅

2. Consider the following set:

U = {all students}

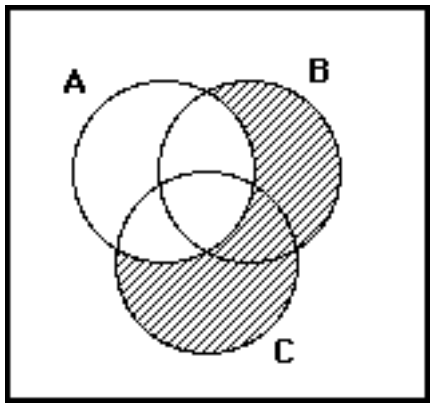
A = {all female students}

B = {all students under 21 years of age}

(A ∪ B)' is the set

(a) {students who are male or at least 21 years old}
(b) {male students who are at least 21 years old}
(c) {male students who are under 21 years old}
(d) {female students who are under 21 years old}
(e) {students who are female or under 21 years old}

3. Identify the shaded region in the following Venn diagram:



(a) $(\mathbf{A \cap B') \cap C}$ (b) $\mathbf{(A \cup B) \cup C}$ (c) $\mathbf{A \cap B \cap C}$

(d) $\mathbf{(B \cup C) \cap A'}$ (e) $\mathbf{(A \cup C) \cap B}$

4. A survey of 120 bank customers revealed that 70 had a checking account, 53 had a savings account, 18 had a savings account and a loan, 17 had a checking account and a loan, 48 had only a checking account and 10 had a checking account, a savings account and a loan. Each customer had at least a savings account, or a checking account, or a loan. The number of customers who had a loan is

(a) 37 (b) 34 (c) 30 (d) 15 (e) 14

5. If **R** and **S** are finite subsets of a universal set **U**, such that

$$\mathbf{n(R') = 20,\ n(S) = 15,\ n(S \cup R') = 25 \text{ and } n(U) = 40}$$

how many elements are there in $\mathbf{S \cap R}$?

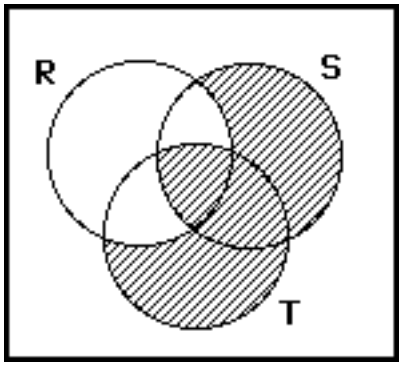
(a) 0 (b) 5 (c) 10 (d) 20 (e) 15

6. An exam contains 19 multiple-choice questions, each having 5 possible answers. In how many different ways can the exam be answered? (Assume that every question must be answered.)

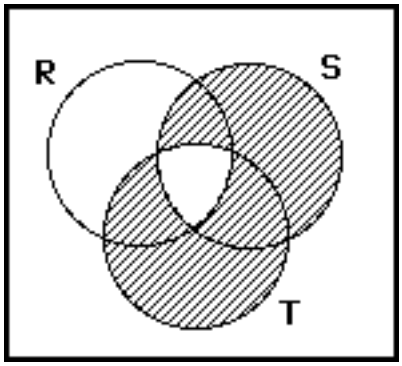
(a) 19! (b) $\binom{19}{5}$ (c) 95 (d) 19^5 (e) 5^{19}

7 In which Venn diagram does the shaded portion represent **(R' ∪ S) ∩ T** ?

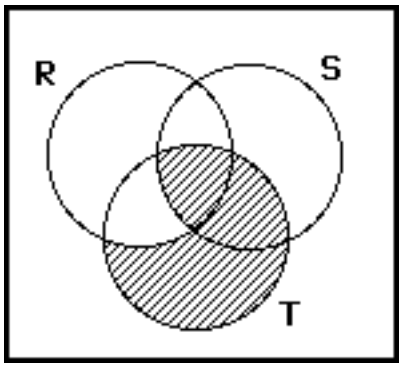
(a)



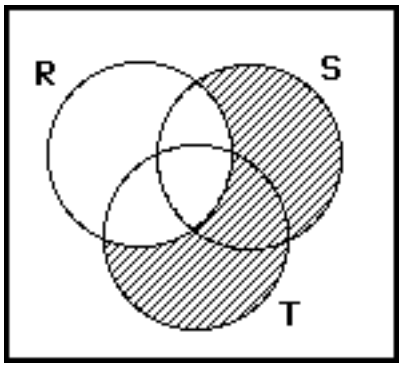
(b)



(c)



(d)



(e) none of the above

8. A set **X** has exactly 7 elements. How many distinct subsets of **X** have at most two elements?

(a) P(7,2) (b) 2^7 (c) 2^7 - C(7,0) - C(7,1) - C(7,2) (d) C(7,2)

(e) 29

9. How many 3-digit numbers can be formed using the digits

{ 1, 2, 3, 4, 5, 6, 7, 8 }

if no repetitions are allowed?

(a) P(8,3) (b) P(9,3) (c) C(8,3) (d) 3^8 (e) 8 !

10. A list of food preferences of 60 species of birds is included when you buy a bird feeder. The list states that 35 species like sunflower seed, 25 like millet, 10 like thistle seed. 15 like both sunflower seed and millet, 5 like both sunflower seed and thistle seed, while 4 like both millet and thistle seed. Further, 3 species like all three of the above types of food. How many like none of the above types of food?

(a) 7 (b) 8 (c) 9 (d) 10 (e) 11

11. What is the coefficient of $\mathbf{x^3y^7}$ in the expansion of $\mathbf{(x + y)^{10}}$?

(a) 56 (b) 84 (c) 90 (d) 220 (e) 120

12. A bag contains 10 blue marbles and 8 red marbles. In how many ways can five marbles be chosen from the bag, without replacement, so that 3 of the chosen marbles are blue and 2 are red?

(a) $\binom{3}{2}$ (b) $\binom{18}{5}$ (c) $10 \bullet 8$

(d) $\binom{10}{3} \bullet \binom{8}{2}$ (e) P(18,5)

13. Which of the following is a valid probability distribution for the sample space **S** = {x, y, z } ?

(a) Pr(x) = 0.7 Pr(y) = 0.0 Pr(z) = 0.2

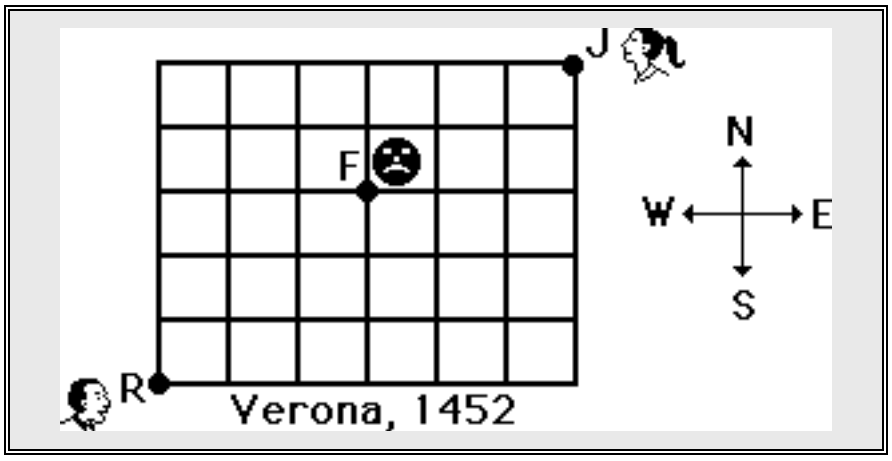
(b) Pr(x) = 0.2 Pr(y) = 0.2 Pr(z) = 0.6

(c) Pr(x) = 0.8 Pr(y) = 0.4 Pr(z) = –0.2

(d) Pr(x) = 0.6 Pr(y) = 0.1 Pr(z) = 0.2

(e) none of the above

14. Here is a street map of Verona in 1452. Romeo is at **R**, Juliet is with her aunt at **J**. The house of Juliet's father is at **F**. In how many ways can Romeo reach Juliet, making sure first that her father is asleep in his house, if Romeo travels North and East only?



(a) $\binom{11}{5} - \binom{6}{3} \cdot \binom{5}{2}$ (b) $\binom{11}{5}$ (c) $\binom{6}{3} \cdot \binom{5}{2}$

(d) $\binom{11}{6}$ (e) 5^6

15. How many subsets of the set {1, 2, 3, 4, 5, 6, 7 } contain no even digit?

(a) $\binom{7}{4}$ (b) P(7,3) (c) $\frac{7!}{4!}$ (d) 2^4 (e) 8

16. A red die and a green die are tossed and the numbers on the uppermost sides are observed. What is the probability that the numbers add up to 6 ?

(a) $\frac{5}{36}$ (b) $\frac{8}{36}$ (c) $\frac{7}{36}$ (d) $\frac{9}{36}$ (e) $\frac{6}{36}$

17. A foundation wishes to award one grant of $ 100,000, three grants of $ 10,000 each and four grants of $ 5,000 each, and three grants of $ 2,000 each. The list of recipients has been already narrowed to 8 recipients. In how many different ways can the awards be made?

(a) 84 (b) 280 (c) 5,040 (d) 8 ! (e) 200

18. An experiment consists of observing the color and make of cars in a dealer's lot. Let

E be the event "the car is red"
F be the event "the car is a Honda"
G be the event "the car is white or a Toyota"
H be the event "the car is a Toyota or a Ford"

Which of the following pair of events are mutually exclusive?

(a) E and F (b) E and G (c) F and H
(d) G and H (e) F and G

19. A digit is selected at random from the digits {1, 2, 3, 4, 5, 6, 7, 8, 9 } . What is the probability that the digit is even or greater than 6?

(a) $\frac{2}{3}$ (b) $\frac{4}{9}$ (c) $\frac{1}{2}$ (d) $\frac{5}{9}$ (e) $\frac{4}{3}$

20. See Cover sheet (the one to be handed in) for a description of this question.