**NAME:** ___________________________________

## Exam 4 for Math 104, Spring 1998

1. State and perform the next elementary row operation when the Gaussian elimination method is applied to the system $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$ (6 points)

2. Solve $\begin{cases} x + z = 1 \\ 2x + y = 1 \\ y - z = -2 \end{cases}$ using the Gaussian elimination method. (10 points.)

3. The result of pivoting the matrix $\begin{bmatrix} 1 & 3 & 5 \\ 5 & \underline{6} & 2 \end{bmatrix}$ about the underlined element is:

   (5 points.)

   a) $\begin{bmatrix} -3/2 & 0 & 4 \\ 5/6 & 1 & 1/3 \end{bmatrix}$

   b) $\begin{bmatrix} 1 & 3 & 5 \\ 5/6 & 1 & 1/3 \end{bmatrix}$

   c) $\begin{bmatrix} 1 & 3 & 5 \\ 3 & 0 & -8 \end{bmatrix}$

   d) $\begin{bmatrix} 1 & 0 & -8/3 \\ 0 & 1 & 23/9 \end{bmatrix}$

   e) none of the above

4. While solving a system of linear equations with the unknowns $x, y$, and $z$ using the Gaussian elimination method, the following matrix was obtained

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 2 \end{array}\right]$$

What can be concluded about the solution of the system? (5 points)

5. Which of the following calculations can be performed? (5 points)

I. $\begin{bmatrix} 4\,0\,2 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$

II. $\begin{bmatrix} 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

III. $\begin{bmatrix} 5 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

IV. $\begin{bmatrix} 2 & 1 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

a) I and III only

b) III and IV only

c) III only

d) IV only

e) I, II, III, IV

6. If B is a $4 \times 2$ matrix and A is a $3 \times 4$ matrix, then the size of A×B is: (5 points)

a) $2 \times 3$

b) $3 \times 2$

c) $4 \times 4$

d) $4 \times 4$

e) none of the above

7. The system $\left\{\begin{array}{rcrrcr} x & - & 2y & +z & = & 4 \\ 3x & + & y & +z & = & 1 \\ x & + & y & +2z & = & 0 \end{array}\right.$ is equivalent to the matrix equation: (5 points)

a) $\begin{bmatrix} 1 & 3 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$

d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & -2 & 1 \\ 3 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$

e) none of the above

8. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (8 points)

9. Given that the matrices A = $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ and B = $\begin{bmatrix} 5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ are inverses of each other, the solution $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ of the system $\begin{cases} 5x - 2y - 2z = 1 \\ -x + y = 2 \\ -x + z = -3 \end{cases}$ is (5 points)

a) $\begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

b) $\begin{bmatrix} 5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \begin{bmatrix} 5 & -2 & -2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

d) none of the above

10. Using the Gauss-Jordan method, find the inverse of the matrix (10 points)

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

11. Consider the matrices:

I. $\begin{bmatrix} 1/6 & 5/3 \\ 5/6 & -2/3 \end{bmatrix}$ II. $\begin{bmatrix} 1 & .7 \\ 0 & .3 \end{bmatrix}$ III. $\begin{bmatrix} .3 & .7 \\ .1 & .9 \end{bmatrix}$

IV. $\begin{bmatrix} .4 & 0 & 1 \\ .6 & .2 & 0 \\ 0 & .8 & 0 \end{bmatrix}$ V. $\begin{bmatrix} .2 & .5 \\ .1 & .1 \\ .7 & .4 \end{bmatrix}$

Which of them are stochastic matrices? (5 points)

a) II and IV only

b) II, III and IV only

c) all except I

d) all except V

e) none of the above

12. A Markov process has the transition matrix A $= \begin{bmatrix} .9 & .4 \\ .1 & .6 \end{bmatrix}$ and the initial distribution matrix $\begin{bmatrix} .5 \\ .5 \end{bmatrix}$. Find the first AND SECOND distribution matrices. (8 points)

13. Which of the following are regular stochastic matrices: (5 points)

I. $\begin{bmatrix} .4 & .5 \\ .6 & .2 \end{bmatrix}$ II. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ III. $\begin{bmatrix} .2 & 0 & .3 \\ .1 & 1 & .4 \\ .7 & 0 & .3 \end{bmatrix}$

IV. $\begin{bmatrix} .1 & .8 \\ .9 & .2 \end{bmatrix}$ V. $\begin{bmatrix} 0 & .6 \\ 1 & .4 \end{bmatrix}$

a) IV only

b) I and IV only

c) II and III only

d) IV and V only

e) II, III and V only

14. Consider the regular stochastic matrix $\begin{bmatrix} .3 & .2 & .8 \\ .3 & .6 & .1 \\ .4 & .2 & .1 \end{bmatrix}$. In order to find its stable distribution, it is necessary to solve the system of equations

a) $\begin{cases} x + y + z = 0 \\ .3x + .3y + .4z = 1 \\ .2x + .6y + .2z = 1 \\ .8x + .1y + .1z = 1 \end{cases}$

b) $\begin{cases} x + y + z = 1 \\ .3x + .2y + .8z = 0 \\ .3x + .6y + .1z = 0 \\ .4x + .2y + .1z = 0 \end{cases}$

c) $\begin{cases} x + y + z = 1 \\ .3x + .3y + .4z = x \\ .2x + .6y + .2z = y \\ .8x + .1y + .1z = z \end{cases}$

d) $\begin{cases} x + y + z = 1 \\ .3x + .2y + .8z = x \\ .3x + .6y + .1z = y \\ .4x + .2y + .1z = z \end{cases}$

e) none of the above

15. Consider the following stochastic matrix.

| | A | B | C | D | E |
|---|---|---|---|---|---|
| A | 0 | 1 | 0 | .2 | 0 |
| B | 1 | 0 | .2 | .3 | 0 |
| C | 0 | 0 | .3 | .1 | 0 |
| D | 0 | 0 | .4 | .2 | 0 |
| E | 0 | 0 | .1 | .2 | 1 |

a) List all the absorbing states. (4 points)

b) Is the matrix an absorbing stochastic matrix? Give a short (1 or 2 sentence ) explanation. (4 points)

16. The stable matrix for the absorbing matrix $\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1/8 \\ 0 & 0 & 3/8 \end{bmatrix}$ is: (5 points)

a) $\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1/8 \\ 0 & 0 & 3/8 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1/8 \\ 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 4/5 \\ 0 & 1 & 1/5 \\ 0 & 0 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 1/5 \\ 0 & 1 & 4/5 \\ 0 & 0 & 0 \end{bmatrix}$

e) none of the above