

1. Consider the following sets.

$$U = \{ \text{all professors} \}$$

$$A = \{ \text{female professors} \}$$

$$B = \{ \text{professors under 40 years of age} \}$$

$A' \cap B$  is the set:

- a) { male professors under 40 years of age }
- b) { male professors who are 40 or older }
- c) { professors who are male or under 40 }
- d) { female professors under 40 years of age }
- e) none of the above

2. In which Venn Diagram does the shaded portion represent  $(A \cup B)'$  ?

3. Out of 30 job applicants, 11 are female, 17 are college graduates, 7 are bilingual, 3 are female college graduates, 2 are bilingual women, 6 are bilingual college graduates, and 2 are bilingual female college graduates.

The number of male college graduates is

- a) 4
- b) 10
- c) 14
- d) 18
- e) none of the above

4. A college student wants to select one foreign language course from among 4 possible courses, one math course from among 7 possible courses, and one economics course from among 5 possible courses. In how many different ways can she select the three courses?

5. How many poker hands consist of 4 clubs and a card of a different suit?

a)  $39 \times C(13, 4)$

b)  $39 \times C(52, 4)$

c)  $48 \times C(13, 4)$

d)  $48 \times C(52, 4)$

e) none of the above

6. If E and F are mutually exclusive and  $\Pr(E) = .2$  and  $\Pr(F) = .6$ , then  $\Pr(E \cup F)$  is:

a) 0

b) .8

c) .12

d) 1

e) none of the above

7. Data was collected about four crimes: robbery, assault, rape and murder. The number of times each time was reported is displayed below.

Crime	Frequency
Robbery	30
Assault	25
Rape	7
Murder	5

What is the probability that one of the four reported crimes is robbery?

8. Let E and F be events such that  $\Pr(E') = 0.4$ ,  $\Pr(F) = 0.3$ , and  $\Pr(E \cup F) = 0.7$ . Compute  $\Pr(E \cap F)$ .

9. An urn contains six red balls and four green balls. A sample of seven balls is selected at random. Find the probability that five red and two green balls are selected.
10. Enrollment statistics at a certain college show that 45% of all students are men, 10% of the student body consists of women majoring in business administration, and 35% of all students major in business administration. A student is selected at random. What is the probability that the selected student majors in business administration if the selected student is a woman?
11. An election between two candidates is held in two districts. The first district, which has 60% of the voters, votes 40% for candidate I and 60% for candidate II. The second district, with 40% of the voters, votes 60% for candidate I and 40% for candidate II. Who wins?
12. In a factory, assembly lines I, II and III produce 60%, 30% and 10% of the total output, respectively. One percent of line I's output is defective, 2% of line II's output is defective and 3% of line III's output is defective. An item is chosen at random. What is the probability that the selected item is defective?

13. Which of the following can be a probability distribution for the random variable X?

a)	<table><thead><tr><th>k</th><th>Pr(X = k)</th></tr></thead><tbody><tr><td>-2</td><td>1/3</td></tr><tr><td>0</td><td>5/12</td></tr><tr><td>1</td><td>1/4</td></tr></tbody></table>	k	Pr(X = k)	-2	1/3	0	5/12	1	1/4	b)	<table><thead><tr><th>k</th><th>Pr(X = k)</th></tr></thead><tbody><tr><td>1</td><td>1/3</td></tr><tr><td>2</td><td>-1/6</td></tr><tr><td>3</td><td>5/6</td></tr></tbody></table>	k	Pr(X = k)	1	1/3	2	-1/6	3	5/6
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0	1/6																		
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2	2/3																		

e) none of the above

14. Suppose that the probability that a federal income tax return contains an arithmetic error is .2. If 10 federal income tax returns are selected at random, the probability that fewer than two of them contain an arithmetic error is

a) .20

b)  $(.8)^{10} + 10(.2)(.8)^9$

c)  $1 - [(.8)^{10} + 10(.2)(.8)^9]$

d)  $(.8)^{10} + (.2)^{10}$

e) none of the above

15. Determine the expected value of the random variable X whose probability distribution is given below.

k	Pr(X = k)
0	.2
1	.4
2	.3
3	.1

16. A certain probability distribution has mean 100 and variance 5. The standard deviation is

a)  $\sqrt{5}$

b) 20

c) 25

d) 500

e) none of the above

17. The IQ of adults in a certain large population is normally distributed with mean 100 and standard deviation 10. If a person is chosen at random from this group, the probability that the person's IQ is less than 85 or greater than 110 is

- a) .0919
- b) .2255
- c) .7745
- d) .9081
- e) none of the above

18. Suppose that a fair coin is tossed 100 times. Find the probability of observing at least 60 heads.

19. The result of performing the elementary row operation  $[3]+(5)[2]$  on the system  $\left[ \begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 0 & 1 & -3 & 2 \\ 0 & -5 & 4 & 1 \end{array} \right]$

is:

a)  $\left[ \begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 0 & 0 & -3 & 2 \\ 0 & -5 & 4 & 1 \end{array} \right]$

d)  $\left[ \begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 0 & -5 & -11 & 11 \\ 0 & -5 & 4 & 1 \end{array} \right]$

b)  $\left[ \begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & -11 & 11 \end{array} \right]$

e) none of the above

c)  $\left[ \begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 0 & -5 & -11 & 11 \\ 0 & -5 & 4 & 1 \end{array} \right]$

20. The system  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & -3 & 5 \\ 1 & 1 & -3 & 4 \end{array} \right]$  is equivalent to the system

a)  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & -3 & 5 \\ 0 & 1 & -3 & 4 \end{array} \right]$                       d)  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & -3 & 5 \\ 0 & 0 & 0 & 4 \end{array} \right]$

b)  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 3 & -9 & 5 \\ 1 & 1 & -3 & 4 \end{array} \right]$                       e) ALL of the above

c)  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & -3 & 5 \\ 0 & 1 & -3 & 6 \end{array} \right]$

21. Find the inverse of the matrix  $\begin{bmatrix} 2 & 1 \\ 3 & 1 \end{bmatrix}$ .

22. The matrices  $\begin{bmatrix} 5 & 2 & -2 \\ 2 & 1 & -1 \\ -3 & -1 & 2 \end{bmatrix}$  and  $\begin{bmatrix} 1 & -2 & 0 \\ -1 & 4 & 1 \\ 1 & -1 & 1 \end{bmatrix}$  are inverses of each other. Use this fact to solve the system

$$\begin{cases} x - 2y & = 5 \\ -x + 4y + z & = 1 \\ x - y + z & = 2 \end{cases}$$

23. Let  $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ , and  $B = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$ .

The entry in the second row, first column of  $A \times B$  is

- a) 6
- b) 8
- c) 18
- d) 22
- e) 34

24. Suppose that the people in a certain city are catching cold. It is observed that after one week, 40% of the people who were sick are still sick. Of the people who were well, 30% are sick after one week.

The transition matrix is

- a) 
$$\begin{array}{c} S \\ W \end{array} \begin{array}{cc} S & W \\ \left[ \begin{array}{cc} .4 & .3 \\ 0 & 0 \end{array} \right] \end{array}$$
- b) 
$$\begin{array}{c} S \\ W \end{array} \begin{array}{cc} S & W \\ \left[ \begin{array}{cc} .3 & .4 \\ .7 & .6 \end{array} \right] \end{array}$$
- c) 
$$\begin{array}{c} S \\ W \end{array} \begin{array}{cc} S & W \\ \left[ \begin{array}{cc} .4 & .3 \\ .6 & .7 \end{array} \right] \end{array}$$
- d) 
$$\begin{array}{c} S \\ W \end{array} \begin{array}{cc} S & W \\ \left[ \begin{array}{cc} .4 & .6 \\ .3 & .7 \end{array} \right] \end{array}$$
- e) none of the above

25. The stable distribution for the regular stochastic matrix  $\begin{bmatrix} .4 & .2 \\ .6 & .8 \end{bmatrix}$  is

- a)  $\begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$
- b)  $\begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$
- c)  $\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$
- d)  $\begin{bmatrix} 3/4 & 3/4 \\ 1/4 & 1/4 \end{bmatrix}$
- e) none of the above

26. The stable matrix for the absorbing stochastic matrix  $\begin{bmatrix} I & S \\ 0 & R \end{bmatrix}$  is

- a)  $\left[ \begin{array}{c|c} I & S(I - R^{-1}) \\ \hline 0 & 0 \end{array} \right]$
- b)  $\left[ \begin{array}{c|c} I & S(R - I)^{-1} \\ \hline 0 & 0 \end{array} \right]$
- c)  $\left[ \begin{array}{c|c} I & S(I - R)^{-1} \\ \hline 0 & R \end{array} \right]$
- d)  $\left[ \begin{array}{c|c} I & S(I - R)^{-1} \\ \hline 0 & 0 \end{array} \right]$
- e) none of the above

27.  $\begin{bmatrix} 2 & 3 \\ 9 & 8 \\ 4 & 7 \end{bmatrix}$  is the payoff matrix of a strictly determined game. A saddle point is
- 2
  - 3
  - 8
  - 9
  - none of the above
28. Suppose that a game has payoff matrix  $\begin{bmatrix} 1 & -2 & 5 \\ 3 & 4 & 3 \end{bmatrix}$ . The optimal pure strategies for R and C are
- R - row 1, C - column 1
  - R - row 2, C - column 1
  - R - row 1, C - column 2
  - R - row 2, C - column 2
  - none of the above
29. Suppose a game has payoff matrix  $\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ , R plays  $[.7 \ .3]$  and C plays  $\begin{bmatrix} .4 \\ .6 \end{bmatrix}$ . What is the expected value?
30. Suppose that a game has payoff matrix  $\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 0 \end{bmatrix}$  and R plays  $[.9 \ .1]$ . Which of  $\begin{bmatrix} .2 \\ .3 \\ .5 \end{bmatrix}$  and  $\begin{bmatrix} .7 \\ .2 \\ .1 \end{bmatrix}$  are better for C?