

1. Let $U = \{\text{the ten decimal digits}\}$ and let
 $R = \{\text{odd digits}\}$
 $S = \{\text{positive digits less than 9}\}$
 $T = \{0, 2, 3, 5, 7, 8\}$

Which of the sets below is $(R \leftrightarrow T) \approx S'$?

- (a) $\{1, 3, 7, 9\}$ (b) $\{1, 9\}$ (c) $\{0, 9\}$ (d) $\{0, 1, 9\}$
(e) \square

2. Consider the following set:

$U = \{\text{all U.S. hotels}\}$

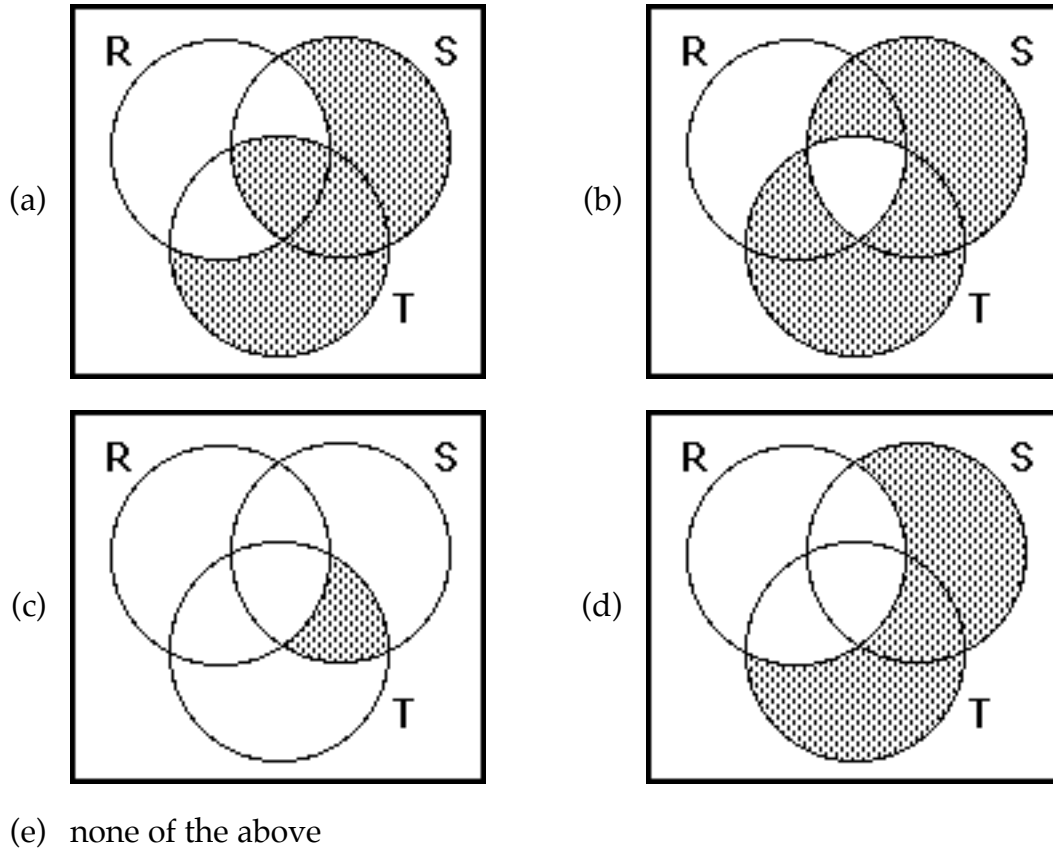
$A = \{\text{U.S. hotels with a swimming pool}\}$

$B = \{\text{U.S. hotels with a restaurant}\}$

then $(A' \approx B)'$ is the set of

- (a) $\{\text{U.S. hotels where you can swim but not eat}\}$
(b) $\{\text{U.S. hotels where you can eat but not swim}\}$
(c) $\{\text{U.S. hotels where you can swim and eat}\}$
(d) $\{\text{U.S. hotels where you can neither swim nor eat}\}$
(e) none of the above

3. In which Venn diagram does the shaded portion represent $R' \leftrightarrow S \leftrightarrow T$?



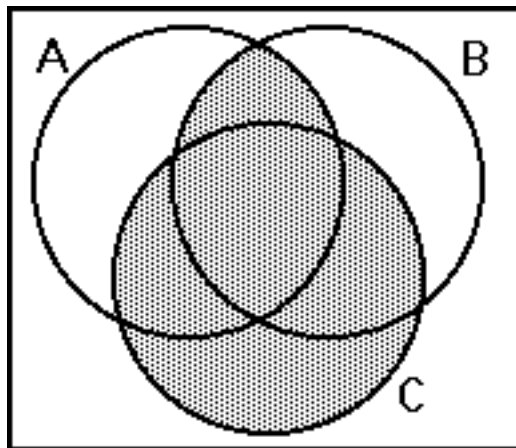
4. 1000 restaurant patrons were surveyed as to their eating habits with regard to Italian, Mexican and Chinese food. The survey revealed that 60% liked Italian food, 51% liked Chinese food, and 50% liked Mexican food. Additionally, 15% liked all three, 32% liked both Italian and Mexican, 33% liked both Italian and Chinese, 10% liked exclusively Mexican food. How many of the patrons disliked all three ethnic types of food?

- (a) 100 (b) 120 (c) 140 (d) 160 (e) none of these

5. An exam contains 6 multiple-choice questions, each having 5 possible answers. In how many different ways can the exam be answered? (Assume that every question must be answered.)

- (a) $5!$ (b) $\binom{11}{5}$ (c) 5^6 (d) 6^5 (e) none of these

6. Identify the shaded region in the following Venn diagram:



- (a) $(A \cap B)' \leftrightarrow C$ (b) $(A \cap B) \leftrightarrow C$ (c) $A \leftrightarrow B \leftrightarrow C$
 (d) $(A \cap B) \leftrightarrow C'$ (e) $(A \leftrightarrow B) \cap C$

7. If R and S are finite subsets of a universal set U , such that

$$n(R) = 15, n(S) = 15, n(S' \leftrightarrow R') = 15 \text{ and } n(U) = 35$$

how many elements are there in $S \leftrightarrow R$?

- (a) 20 (b) 10 (c) 15 (d) 5 (e) 0

8. A list of food preferences of 50 species of fish is included when you buy a fish tank. The list states that 31 species like crushed seaweed, 20 like insect powder, 12 like live plankton. 10 like both crushed seaweed and insect powder, 6 like both crushed seaweed and live plankton, 4 like both insect powder and live plankton. Also, 2 species like all three of the above types of food. How many like none of the above types of food?

(a) 15 (b) 10 (c) 0 (d) 7 (e) 5

9. A set X has exactly 9 elements. How many distinct subsets of X have two or more elements?

(a) $P(9,2)$ (b) $9! - 1$ (c) $2^9 - C(9,0) - C(9,1)$ (d) $C(9,2)$
(e) $6! - 1! - 0!$

10. A poker hand consists of five cards selected from a deck of 52 cards. How many poker hands consist of four cards from one suit (either spades, hearts, diamonds or clubs) and a card from a different suit? (*In poker parlance this is a busted flush*)

- (a) $4 \sum \binom{13}{4} \sum 39$ (b) $39 \sum \binom{52}{4}$ (c) $48 \sum \binom{13}{4}$
 (d) $48 \sum \binom{52}{4}$ (e) none of these

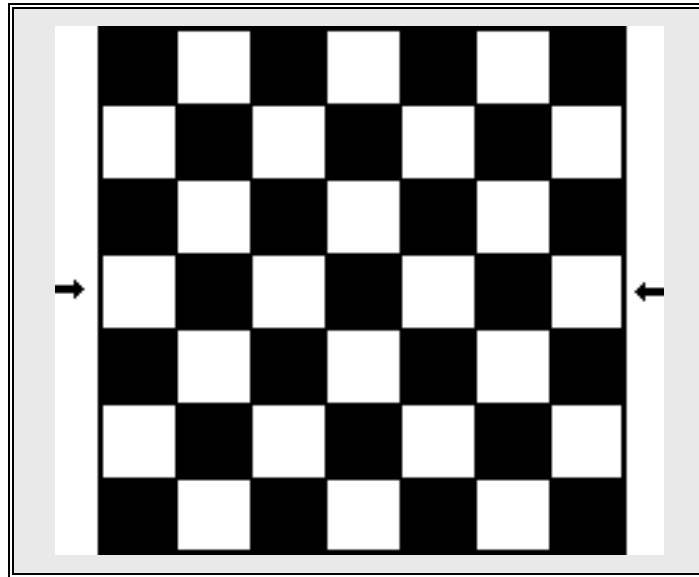
11. What is the coefficient of a^6b^3 in the expansion of $(a + b)^9$?

- (a) 48 (b) 1 (c) 84 (d) 9 (e) 72

12. A sample (without replacement) of three cookies is picked from a bag containing three chocolate chip cookies and five sugar almond cookies. How many such samples consist of exactly one chocolate chip and two sugar almond cookies?

- (a) $\binom{5}{2}$ (b) $\binom{8}{3} - 5$ (c) $\frac{\binom{8}{3}}{2!}$
 (d) $\binom{3}{1} \sum \binom{5}{2}$ (e) $\binom{3}{1} + \binom{5}{2}$

13. On the 7x7 checkerboard square shown below, a "move" is a step from any one square to one next to it in a diagonal direction (no North-South nor East-West moves are allowed, nor is jumping squares allowed.) You are on the white square on the left identified by the arrow and wish to reach the white square on the right identified by the arrow in the smallest possible number of moves. How many moves are needed, and how many distinct paths are available to you?



- (a) 6 moves, 10 paths (b) 6 moves, 20 paths (c) 7 moves, 30 paths
 (d) 7 moves, $\binom{7}{3}$ paths (e) 6 moves, 2^6 paths

14. Let $S = \{x, y, w, z\}$ be a sample space. Which of the following is a valid probability distribution for S ?

- (a) $\Pr(x) = 0.6$ $\Pr(y) = 0$ $\Pr(z) = 0.3$ $\Pr(w) = 0.1$
 (b) $\Pr(x) = 0.65$ $\Pr(y) = 0$ $\Pr(z) = 0.15$ $\Pr(w) = 0.15$
 (c) $\Pr(x) = 0.55$ $\Pr(y) = 0.15$ $\Pr(z) = -0.3$ $\Pr(w) = 0.1$
 (d) $\Pr(x) = 0.6$ $\Pr(y) = 0.15$ $\Pr(z) = 0.2$ $\Pr(w) = 1.1$
 (e) none of the above

15. How many subsets of the set {E, X, A, M, S } do not contain a vowel?

- (a) $\binom{5}{2}$ (b) $P(5,2)$ (c) $\frac{5!}{2!}$ (d) 24 (e) 8

16. An experiment consists of observing the sex and race of 150 students in an anatomy class at Diverse Medical Center. Let

E be the event "the student is a white male"

F be the event "the student is either female or white"

G be the event "the student is either male or black"

H be the event "the student is either hispanic or black"

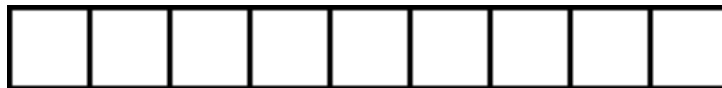
Which of the following pair of events are mutually exclusive?

- (a) E and F (b) E and G (c) G and H
(d) E and H (e) F and H

17. Suppose a red die and a green die are tossed and the numbers on the uppermost sides are observed. What is the probability that the numbers add up to 8 ?

- (a) $\frac{7}{36}$ (b) $\frac{6}{36}$ (c) $\frac{5}{36}$ (d) $\frac{4}{36}$ (e) $\frac{3}{36}$

18. You have to paint the nine squares shown below as follows: three must be red, three green, two yellow and one blue. How many different patterns can you produce?



- (a) 84 (b) 36 (c) 213 (d) 5,040 (e) 72

19. A digit is selected at random from the digits $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. What is the probability that the digit is either more than 5 or even?

(a) $\frac{2}{9}$

(b) $\frac{2}{3}$

(c) $\frac{1}{2}$

(d) $\frac{4}{9}$

(e) $\frac{4}{3}$

20. The odds in favor of a vowel in my instructor's last name are:

(a) 3 to 3

(b) 3 to 4

(c) 3 to 5

(d) 3 to 6

(e) 3 to 7