

1. Here is a game which you are to play against me. You toss one fair die, and if the top face shows 1 or 6 I pay you \$ 11.00 (eleven dollars). If the top face shows anything else you toss a fair coin and, if the coin shows 'heads' I pay you the amount (in dollars) shown on the face of the die. If the coin shows 'tails' you pay me twice the amount (in dollars) shown on the face of the die. *For example,* if you toss a five and then 'heads' I pay you \$ 5.00 (five dollars), if you toss a four and then 'tails' you pay me \$ 8.00 (eight dollars), if you toss a one or a six I pay you \$ 11.00 (eleven dollars.) Who should pay whom how much to play this game and make it fair? (That is, neither in my favor nor in yours.) The table below shows your possible earnings in dollars

-4.00	-6.00	-8.00	-10.00	2.00	3.00	4.00	5.00	11.00

- (a) you should pay me \$ 2.50 (b) I should pay you \$ 2.00
 (c) you should pay me \$ 1.50 (d) I should pay you \$ 1.00
 (e) none of the above

2. The following is known about a random variable X:

$E(X^2) = 6$ and also $Var(X) = E(X)$

Then $Var(X)$ equals

- (a) 5 (b) 4 (c) 3 (d) 2 (e) 1

3. Compute the variance of the random variable X whose probability distribution is shown in the table below

k	$P(X = k)$
-2	0.2
0	0.2
1	0.2
2	0.2
3	0.2

- (a) 2.8 (b) 2.6 (c) 3.6 (d) 2.96 (e) 1.6

4. Compute the variance of the random variable X whose probability distribution is shown in the table below, given that $E(X) = 0$.

k	$P(X = k)$
-2	
-1	0.3
0	0.1
2	
3	0.1

- (a) 2.8 (b) 2.2 (c) 1.8 (d) 2.6 (e) 3.2

Questions 5, 6 and 7 refer to the following situation:

The state of Indiana recently held a 'deer hunt' in Potato Creek State Park to thin out somewhat the deer herd in the park (*I am not making this up.*) They let 50 hunters in the park, with a limit of one deer per hunter per day. You may assume that each hunter has an 80% chance of bagging his/her deer each day, and that the hunters hunt independently of each other (*hopefully they don't bag each other either!*)

5. After three days of hunting, what is the expected number of deer 'thinned out' from the herd?
(a) 80 (b) 90 (c) 100 (d) 110 (e) 120
6. What is the standard deviation of the number of deer thinned out in one day?
(a) $2\sqrt{2}$ days (b) $2\sqrt{2}$ deer (c) $2\sqrt{2}$ hunters (d) 8 (e) 2
7. What is the probability that Ms. Sangrefria, one of the hunters, will bag herself at least two deer during the three day hunt?
(a) 80.6% (b) 90.8% (c) 89.6% (d) 75.4% (e) 99.2%

8. A 36 questions **True/False** questionnaire tests your knowledge of the mating habits of the Katmandu zebra, the only zebra with a polka dot skin pattern. Assume you don't know a thing about the Katmandu zebra and are therefore just guessing. What do you estimate is your probability of getting at least 20 questions answered correctly? (*Only one among the answers shown is the most accurate*)

(a) 40% (b) 35% (c) 30% (d) 25% (e) 20%

9. Here is a game which you are to play against me, but this time it will cost you \$ 1.50 (one dollar fifty cents) to play it once. I hand you a fair coin, and you toss it until you have either thrown two 'heads' (not necessarily consecutive) or you have tossed the coin four times, whichever comes first. I will pay you back \$ 1.00 (one dollar) for every 'tails' you toss. (for example, if you toss no 'tails' you lose a dollar and a half, if you toss one tail only, you lose fifty cents, etc.) What do you expect to win (or lose) per game?

(a) expect to lose 62.5 cents/game (b) expect to lose 12.5 cents/game
(c) expect to win 62.5 cents/game (d) expect to win 12.5 cents/game
(e) expect to break even

10. Let Z be the standard normal random variable. Find a value z_0 such that

$$P(\{Z \geq z_0\} \approx \{Z \leq -z_0\}) = 0.007$$

- (a) $z_0 = 0.0035$
- (b) $z_0 = 2.8$
- (c) $z_0 = 2.7$
- (d) $z_0 = 2.6$
- (e) $z_0 = 2.5$

Questions 11 and 12 refer to the random variable X which denotes the yearly height of snowfall in South Bend, in inches. It is known that X is normally distributed.

11. What is the probability that snowfall in South Bend will be below average in the 1999-2000 winter?

- (a) below average
- (b) above average
- (c) 100%
- (d) 75%
- (e) 50%

12. It is also known that the average snowfall is 90 inches, with standard deviation 4 inches. What is the probability that South Bend will get eight feet or more of snow this winter? (There are 12 inches in one foot.)

- (a) 0.0668
- (b) 0.2499
- (c) 0.1587
- (d) 0.0047
- (e) 0.9999

15. The monthly demand for blood in Transylvania is normally distributed with $\mu = 177$ liters and $\sigma = 20$ liters. How many liters of blood should the blood bank have on hand at the start of any month in order to be 99.6% sure that demand will not exceed supply before the end of that month?

- (a) 180 (b) 230 (c) 190 (d) 210 (e) 240

16. One in ten batteries manufactured by the Sonpillos Co. is defective. GM buys batteries from the company in lots of 100, but the lot is rejected if it is found to contain 9 or more defective batteries. Use the normal approximation to the binomial to estimate the probability that a lot of batteries will be rejected by GM.

- (a) 0.6915 (b) 0.3085 (c) 0.8413 (d) 0.1587 (e) 0.9987

17. For certain types of transistors the amount of hours a transistor will function before requiring replacement is a random variable with mean $\mu = 3,000$ hours and $\sigma = 250$ hours. Suppose that 4,800 such transistors are installed in an orbiting satellite. Estimate the number of transistors that will require replacement between 2,000 and 4000 hours from the time of installation. (Use *Tchebyshev's*)
- (a) $\geq \frac{15}{16}$ (b) $\leq 4,500$ (c) ≥ 313 (d) ≤ 313 (e) $\geq 4,500$

18. Each time a basketball player shoots a personal foul, his probability of making the basket is 0.8 . Let X be the number of baskets he makes out of 10 attempts. What is the standard deviation of X ?
- (a) $\sqrt{1.6}$ (b) 1.6 (c) $\sqrt{1.06}$ (d) 40 (e) $4\sqrt{10}$

19. One hundred and fifty dice are tossed simultaneously, and the number of faces showing 4 or less is observed. What is the expected value of this number?

- (a) 75
- (b) 100
- (c) 120
- (d) 80
- (e) none of the above

20. Two letters are picked at random, simultaneously and without replacement, from your instructor's last name. What is the expected number of vowels?

- (a) $\frac{8}{7}$
- (b) $\frac{7}{7}$
- (c) $\frac{6}{7}$
- (d) $\frac{5}{7}$
- (e) $\frac{4}{7}$