

**Math. 104 - Fall 1999 - Final Examination ..... 1**

1. Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$  be the universe and let

$$A = \{\text{even integers in } U\} ; B = \{1, 2, 3, 4, 5\} ; C = \{\text{multiples of 3 in } U\}$$

Then the set  $(A \leftrightarrow B') \cap (B \leftrightarrow C')$  is

- (a)  $\{3\}$       (b)  $\{1, 2, 4, 5, 6, 8\}$       (c)  $U$       (d)  $\{6, 8\}$   
(e)  $\emptyset$

2. In 1998 the sun shone in South Bend during 123 days (but it may have also rained during some of those days.). In addition, in 1998 it rained on 168 days in South Bend (but on some of those days the sun also shone.) Finally, in 1998 in South Bend there were 99 days during which it did not rain and the sun did not shine all day. During how many days could you have seen a rainbow in South Bend in 1998?

- (a) 23      (b) 24      (c) 356      (d) 68      (e) 25

3. You have 4 apples and 3 bananas in your refrigerator on Sunday night of a certain week. In how many different ways can you eat one piece of fruit per day during the next seven days??

(a) 35      (b) 5,040      (c) 144      (d)  $\frac{7!}{4!}$       (e) 13,612,578

4. How many possible ways are there of lining up 8 different models of Toyondas automobiles for a photograph if 2 of the models (the Linxette and the Validator) must occupy the two middle positions, i.e. #4 and #5?

(a)  $8! - 2 \times 6!$       (b)  $\frac{8!}{2}$       (c)  $2 \times 6!$       (d)  $C(8,6)$       (e)  $P(8,6)$

5. A subcommittee of three persons is picked from a committee consisting of three males and four females. How many such subcommittees consist of exactly two females and one male?

(a)  $\binom{3}{1} \Sigma \binom{4}{2}$       (b)  $\binom{7}{3} - 4$       (c)  $\frac{\binom{7}{3}}{2!}$   
 (d)  $\binom{3}{2} \Sigma \binom{4}{1}$       (e)  $\binom{4}{1}$

6. What is the coefficient of  $p^5q^5$  in the binomial expansion of  $(p + q)^{10}$ ?

- (a) 210            (b) 252            (c) 270            (d) 230            (e) 212

7. At AllaDiavola Pizzeria the basic pizza is made with dough, cheese and tomato sauce. One can also order any combination (or none) of the four toppings listed below:

**sausage            pepperoni    mushrooms    green peppers**

One pizza is chosen at random from an order, and the combination of toppings (including the possible no-topping one) is recorded. How many entries are there in the sample space for this experiment?

- (a) 5            (b) 10            (c) 15            (d) 16            (e) 32

8. Two fair dice are rolled, and the numbers on their top faces are recorded. Consider the following events:

E : the two numbers are the same  
(*gamblers call this 'the hard way'*)

F : the sum of the two numbers is 7 or 11

G : at least one of the two numbers is even

Which of the following statements about these events is true?

- (a) E and F are mutually exclusive
  - (b) E and G are mutually exclusive
  - (c) F and G are mutually exclusive
  - (d) each pair of these events is mutually exclusive
  - (e) no two of these events are mutually exclusive
9. Suppose E and F are independent events. Which of the following statements is NOT true?
- (a) E and  $F'$  are independent
  - (b)  $E'$  and F are independent
  - (c) E and  $E'$  are independent
  - (d)  $E'$  and  $F'$  are independent
  - (e) F and E are independent

10. Suppose that E and F are independent events with  $\Pr(E) = 0.6$  and  $\Pr(F) = 0.5$ .  
What is  $\Pr(E \approx F)$ ?

- (a) 0.74      (b) 0.70      (c) 0.84      (d) 0.76      (e) 0.80

11. A fair coin is tossed three times. What is the probability of obtaining tails at least once, given that at least two heads are obtained?

- (a)  $\frac{1}{2}$       (b)  $\frac{3}{4}$       (c)  $\frac{1}{4}$       (d)  $\frac{1}{8}$       (e) 0

12. Suppose E and F are events with  $P(E) = 0.5$ ,  $P(F) = 0.5$  and  $P(E \approx F) = 0.70$ . Find  $P(E|F)$

- (a) 0.74      (b) 0.70      (c) 0.84      (d) 0.6      (e) 0.30

13. A 12-member jury consists of seven men and five women. At the end of their deliberation they enter the courtroom in single file to announce their verdict. What is the probability that the third juror to enter is the first male juror to enter?
- (a)  $\frac{7}{66}$       (b)  $\frac{47}{1264}$       (c)  $\frac{5}{7}$       (d)  $\frac{20}{132}$       (e)  $\frac{7}{12}$

**THE NEXT TWO QUESTIONS (14 and 15) REFER TO THE FOLLOWING SITUATION:**

In a certain Texas city 40% of the inhabitants prefer Mexican food, 25% prefer Italian food, the rest prefer anything but Mexican or Italian. Of those who prefer Mexican food 25% admit to eating out more often than at home, while the similar percentage for Italian food lovers is 40% , as is for the rest. The table below summarizes the numbers stated.

	<i>Food preference</i>	<i>Proportion eating out more often</i>
Mexican food	0.40	0.25
Italian food	0.25	0.40
Anything but ...		0.40

14. What portion of the inhabitants of this Texas city eat out more often than at home?
- (a) 51%      (b) 70%      (c)  $\frac{17}{50}$       (d) 17%      (e) 140%
15. A randomly chosen person states that she eats out more often than at home. What is the probability that the woman prefers Italian food?
- (a) 0.51 (b) 0.70 (c) 0.21 (d)  $\frac{7}{17}$  (e)  $\frac{5}{17}$

THE NEXT TWO QUESTIONS (16 and 17) REFER TO THE FOLLOWING SITUATION:

The table below shows the number of nickels, dimes, and quarters I have in my pocket today.

nickels	dimes	quarters
12	8	10

16. What is the expected value of a coin pulled at random from my pocket today?  
(a) 5      (b) 13      (c) 10      (d) 20      (e) none of these
17. What is the standard deviation of the value of a coin pulled at random from my pocket today?  
(a) 76      (b) 7.6      (c) 0.76      (d)  $2\sqrt{19}$       (e)  $19\sqrt{2}$

18. An experiment consists of tossing a fair die 10 times and counting the number of even faces obtained. What is the probability that the count will be 1 or 2?

(a)  $\binom{10}{1} + \binom{10}{2}$       (b)  $\frac{55}{4}$       (c)  $\frac{55}{2^{10}}$

(d)  $\left[\binom{10}{0} + \binom{10}{2}\right]\left(\frac{1}{2}\right)^{10}$       (e) 1

19. Buonladro Bakery asserts that the weight of a loaf of Italian bread sold at the bakery is normally distributed with  $\mu = 1,000$  grams and  $\sigma = 30$  grams. Two loaves of bread bought independently at the bakery have weighted less than 955 grams each. Mister Buonladro says it's just a coincidence. What is the probability of this "coincidence"? (*Exactly one of the answers below is the most accurate.*)

(a) 0.0668    (b) 10%    (c) 0.45%    (d) 0.9332    (e) 0.01%



20. A pharmaceutical firm determines that the number of defective vials of vaccine in each batch of 1000 vials it produces is a random variable with mean 20 and variance 25. Use Tchebyshev's to estimate the probability that one batch of 1000 vials will have more than 40 defective vials. (*Exactly one of the answers below is the most accurate.*)

- (a)  $\leq 0.06$     (b)  $\geq 0.06$     (c)  $< 63$     (d)  $\geq 63$     (e)  $\geq 41$

21. Scemo lottery tickets on the island state of Noh-math cost \$1.00 each. Winning tickets pay \$ 5.00 (you actually win only four dollars.) The government advertises that, on the average, there is one winning ticket among every 10 tickets sold. You buy 100 Scemo lottery tickets. Estimate the probability that you will at least break even.

- (a) 0.27    (b) 0.0008    (c) 0.9992    (d) 0.0004    (e)  $\frac{20}{100}$

**THE NEXT TWO QUESTIONS (22 and 23) REFER TO THE FOLLOWING SITUATION:**

A random sample of size 10 of a certain random variable  $X$  has yielded the following numbers:

2      5      3      4      6      2      3      3      6      6

22. Estimate the mean of the random variable  $X$ .

- (a) 0.4      (b) 10      (c) 1      (d) 4      (e) 1

23. What is the sample variance of this sample?

- (a) 2.4      (b) 8      (c)  $\frac{8}{9}$       (d)  $\frac{2\sqrt{6}}{3}$       (e)  $\frac{24}{9}$

