Math 104
Midterm 3, April 7

1. Each of three people randomly chooses one of three calculus sections to take $(A, B$, or $C$ ).
(a) [10 points] What is the probability that they all choose the same one?
(b) $[10$ points $]$ What is the probability that they each choose a different section?
2. Let $S$ be a sample space and $E$ and $F$ events associated with $S$. Suppose that $\operatorname{Pr}(E)=0.5, \operatorname{Pr}(F)=0.3$, and $\operatorname{Pr}(E \cap F)=0.1$.
(a) [5 points] Calculate $\operatorname{Pr}(E \mid F)$ and $\operatorname{Pr}(F \mid E)$.
(b)[5 points] Are $E$ and $F$ independent events? Explain.
(c) Calculate $\operatorname{Pr}\left(E \mid F^{\prime}\right)$.
(d) Calculate $\operatorname{Pr}\left(E^{\prime} \mid F^{\prime}\right)$.
3. (a)[5 points] State De Morgan's Laws.
(b)[5 points] State Complement Rule.
(c)[5 points] State Inclusion-Exclusion Principle (the one for probability, not for set and counting).
(d)[5 points] Show that if events $E$ and $F$ are indepedent of each other, so are $E^{\prime}$ and $F^{\prime}$.
4. (a)[5 points] State Product Rule.
(b) $[5$ points $]$ Suppose that $E$ and $F$ are two indepedent events and $\operatorname{Pr}(E)=0.3$. What is $\operatorname{Pr}(E \mid F)$ ?
(c)[5 points] Suppose that $E$ and $F$ are two identical events, namely $E=F$. What is $\operatorname{Pr}(E \mid F) ?$
(d)[5 points] Suppose that $F$ and $F$ are two complementary events, namely $E=F^{\prime}$. What is $\operatorname{Pr}(E \mid F)$ ?
5. Suppose that a random variable $X$ has probability distribution given by the following table:

| $k$ | $\operatorname{Pr}(X=k)$ |
| :---: | :---: |
| -1 | 0.2 |
| 0 | 0.3 |
| 1 | 0.1 |
| 2 | 0.4 |

(a)[5 points] What are the possible values of the random variable $X$ ?
(b)[5 points] What are the possible values of the random variable $X^{2}$ ?
(c)[5 points] Determine the probability distribution of the random variable $X^{2}$.
(d)[5 points] Draw the histogram of the probability distribution of $X^{2}$.

