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Math 105: Calculus

Name: \_\_\_\_\_

Exam I

Instructor: \_\_\_\_\_

September 18, 1997

Time of MWF class: \_\_\_\_\_

Calculators are not allowed. Do not remove this answer page— you will return the whole exam. There are 16 multiple choice questions, worth 5 points each, and 3 partial credit questions worth a total of 20 points. Record your answers to the multiple choice problems by placing an  $\times$  through one letter for each problem on this answer sheet. On problems 17–19, write your answer clearly in the place indicated, but also show all necessary work.

**You are taking this exam under the honor code.**

If  $f(x) = \frac{x^2 + x}{x + 1}$  and  $g(x) = \frac{1}{x}$ , find  $f(x) + g(x)$ .  $\frac{x^3 + x^2 + x + 1}{x^2 + x} \frac{x^2 + x + 1}{2x + 1}$

$\frac{x^3 + x^2 + x + 1}{x + 1} \frac{x^3 + x^2 + 1}{2x + 1} \frac{x^2 + x + 1}{x^2 + x}$

Let

$$f(x) = \begin{cases} x^2 + 1, & \text{if } x < 0; \\ 3, & \text{if } 0 \leq x \leq 1; \\ 2x + 1, & \text{if } x > 1. \end{cases}$$

Which of the following is true?

$f(x)$  is continuous everywhere except at  $x = 0$ .  $f(x)$  is continuous everywhere except at  $x = 1$ .  $f(x)$  is continuous everywhere except at  $x = 0$  and  $x = 1$ .  $f(x)$  is continuous everywhere.  $f(x)$  is not a function.

The cost of receiving a certain operation at Mercy Hospital is \$3,000, and the cost for a semiprivate room is \$500 per day. (Ignore all other costs.) Bob received a bill of \$5,000 for receiving the operation and staying  $x$  days in the hospital. What is  $x$ ?

4 2 6 5 3

If  $g(t) = t^2 + 1$  and  $h(t) = \sqrt{t}$ , find  $h(g(t))$ .

$$\sqrt{t^2 + 1} t + 1 \quad (\sqrt{t} + 1)^2 t + 2 t^4 + 1$$

Find all the points of intersection of the curves  $y = x^2 - 3x + 2$  and  $y = -8x + 8$ .

$(-6, 56)$  and  $(1, 0)$   $(-6, 0)$  and  $(1, 0)$   $(1, 0)$  and  $(2, 0)$   $(1, 0)$  and  $(2, -8)$   $(1, 0)$

Which of the following is equal to  $\left(\frac{1}{27}\right)^{-2/3}$  ?

9  $\frac{1}{9}$  81  $\frac{1}{81}$  18

Find  $\lim_{h \rightarrow 0} \frac{\sqrt{25 + h} - 5}{h}$ , if it exists.

$\frac{1}{10}$  10  $\frac{1}{5}$  5 the limit does not exist

Find the equation of the line through the point  $(-2, 3)$  and perpendicular to the line  $x - 2y = 5$ . Make sure your answer is in the “slope-intercept” form,  $y = mx + b$ .

$y = -2x - 1$   $y = -\frac{1}{2}x + 2$   $y = \frac{1}{2}x + 2$   $y = -2x - 4$   $y = -\frac{1}{2}x - 1$

Find the slope of the curve  $y = x^3 - x^2$  at  $x = 2$ .

8 4 1 16 2

Find the equation of the tangent line to the curve  $y = x^2 - \frac{1}{x}$  at  $x = 1$ .

$y = 3x - 3$   $y = 3x$   $y = x - 1$   $y = x + 1$   $y = 2x + 1$

Let  $f(x) = \sqrt{x^2 + x}$ . Find  $f'(2)$ .

$\frac{5}{2\sqrt{6}}$   $\frac{1}{2\sqrt{6}}$   $\frac{5\sqrt{6}}{2}$   $\sqrt{6}$   $\frac{2}{\sqrt{6}}$

-6 0 6 -1 the limit does not exist

Find the  $x$ -coordinate of the point on the graph of  $y = \sqrt{x}$  where the tangent line has slope  $\frac{1}{4}$ .

$4\sqrt{2}$  8 16  $\sqrt{8}$

For the graph of  $y = x^2 - x$ , which of the following represents the slope of the secant line between the point on the graph where  $x = 2$  and the point on the graph where  $x = 2 + h$ ?

$3 + h$   $4 + h$   $2 + h$   $5 + h$   $\frac{2 + h}{h}$

Find the domain of the function  $f(x) = \frac{1}{\sqrt{x+1}}$

$(-1, \infty)$   $[-1, \infty)$   $(1, \infty)$   $(-\infty, 1)$   $(-\infty, -1)$

Which of the following is true about the differentiability of the function

$$f(x) = \begin{cases} x^2 + 1, & \text{if } x < 0; \\ 1 - x^2, & \text{if } 0 \leq x \leq 1; \\ -2x + 2, & \text{if } x > 1. \end{cases}$$

$f(x)$  is differentiable at both  $x = 0$  and  $x = 1$   $f(x)$  is differentiable at  $x = 0$  but not at  $x = 1$   $f(x)$  is differentiable at  $x = 1$  but not at  $x = 0$   $f(x)$  is not differentiable at either  $x = 0$  or  $x = 1$   $f(x)$  is not a function

**Partial credit Part**

17. A farmer wants to enclose his livestock in a rectangular corral with two partitions, forming three sections of equal size as illustrated below.

Suppose that the total area of the corral (i.e. all three sections combined) is 600 square feet. Write your answers to the following questions in the box provided, and show all necessary work in the remaining space.

(a) (3 points) Express  $y$  as a function of  $x$ .

(b) (4 points) Express the total length of fencing as a function of  $x$ .

18. There is a point  $P$  on the graph of  $f(x) = x^2 + x$  where the tangent line to the graph at  $P$  has equation  $y = -5x + b$ , for some constant  $b$ .

(a) (4 points) What is the  $x$ -coordinate of  $P$ ?

(b) (4 points) What is the value of  $b$ ?

19. (5 points) Sketch a line on the graph below which is tangent to the graph and which has slope  $\frac{1}{2}$ .