

Calculators are not allowed. There are 19 multiple choice questions, worth 5 points each, and you get an additional 5 points for writing your name. Record your answers to the multiple choice problems by placing an  $\times$  through one letter for each problem on this answer sheet.

**You are taking this exam under the honor code.**

If  $f(x) = \frac{1}{x^2} + 2x + 5$  compute  $f''(x)$ .

$\frac{6}{x^4} - \frac{6}{x^4} - \frac{2}{x^3} + 2 \quad 2 \quad 6x^4$

Find  $\left. \frac{d^2}{dx^2} (x^4 - 3x^3 + x^2 - x + 24) \right|_{x=0}$ .

$2 \quad 12 \quad -3 \quad 24 \quad 0$

The average rate of change of the function  $f(x) = x^3 - x^2 + 2x + 1$  over the interval from 1 to 3 is

$11 \quad 22 \quad 25 \quad 3 \quad 23$

Find the instantaneous rate of change of the function  $g(t) = \sqrt{3t - 2}$  at  $t = 2$ .

$\frac{3}{4} \quad 2 \quad \frac{3}{2} \quad \frac{3}{8}$  it does not exist

The profit obtained by a manufacturer when producing  $x$  units of some product is given in dollars by the function

$$P(x) = 0.004x^4 + 2x .$$

Find the marginal profit at production level  $x = 10$  units of product.

$18 \quad 16 \quad 21 \quad 420 \quad 250$

The cost of producing  $x$  units of a product is given in dollars by the function

$$C(x) = 0.05x^2 - 3x + 100 .$$

Find the production level  $x$  at which the marginal cost is 10 dollars/unit.

130 units 135 units 126 units 50 units 200 units

The position function of an object moving on a straight line is given by

$$s(t) = 5t^2 + 4\sqrt{t} ,$$

where  $s$  is expressed in miles and  $t$  in hours. What is the velocity of the object at time  $t = 4$  hours?

41 mph 65 mph 35 miles 39 mph 88 miles

Suppose  $T(t)$  is the temperature on a hot summer day at time  $t$  hours. If  $T'(10) = 4$ , by approximately how much will the temperature rise from 10:00 to 10:30 ?

$2 \quad 4 \quad -4 \quad 1 \quad 3$

Which of the following is true for the function  $f$  whose graph is pictured below? (Only one answer is true.)

$f$  is decreasing and concave up on the interval  $(0, 1)$   $f$  is increasing in the interval  $(-1, 3)$   $f$  is concave up in the interval  $(-1, 3)$   $f$  is decreasing and concave down in the interval  $(-1, 0)$   $f$  is increasing and concave up on the interval  $(2, 3)$

The graph shown below is the graph of a function  $f(x)$  near  $x = 2$ . Which of the following statements is true?

$f'(2) > 0$  and  $f''(2) < 0$   $f'(2) < 0$  and  $f''(2) < 0$   $f'(2) = 0$  and  $f''(2) < 0$   $f'(2) < 0$  and  $f''(2) = 0$  none of the above statements is true

For what value(s) of  $x$  does the function

$$f(x) = x^3 + 6x^2 - 36x + 7$$

have a relative minimum point?

$x = 2$   $x = 2$  and  $x = -6$   $x = -6$   $x = -20$   $f$  has no relative minimum point

What is the absolute maximum value of the function  $f(x) = -5x^2 + 10x + 3$ ?

8 -10 3 12  $f$  has no absolute maximum value

For what value(s) of  $x$  does the function  $f(x) = 2x^4 - 12x^2 - 5$  have an inflection point?

$x = -1$  and  $x = 1$   $x = -1$  and  $x = 0$   $x = 0$   $x = 2$  and  $x = -2$   $f$  has no inflection points

Consider the function

$$f(x) = 2x + 3 + \frac{8}{x},$$

defined for  $x > 0$ . Find the absolute minimum value of  $f(x)$ .

11 0 15 7  $f$  has no absolute minimum value

On which of the following intervals is the function  $f(x) = (x - 1)^3$  decreasing?  
 $f$  is never decreasing  $(-\infty, 0)$   $(-\infty, 1)$   $(0, +\infty)$   $(1, +\infty)$   
 Which of the graphs sketched below best represents the graph of the function

$$f(x) = x^3 + \frac{3}{2}x^2 - 6x ?$$

I II III IV V

A ball is thrown straight up in the air and its height in feet (above the ground) after  $t$  seconds is given by

$$h(t) = 6 + 64t - 16t^2 .$$

What is the maximum height reached by the ball?

70 6 65 75 2

A rectangular garden with two sides of length  $x$  and two of length  $y$  (in feet) is to be surrounded by a wooden fence along the sides of length  $x$  and by a wire fence along the sides of length  $y$ . The cost of the wooden fence is 10 dollars per foot and the cost of the wire fence is 5 dollars per foot. Find the maximum area that the garden can have (in sq feet) if the total cost of fencing is 400 dollars.

200 192 220 198 150

An open rectangular box (i.e. no top) has a square base and has a volume of 4 cubic feet. What is the smallest surface area (in sq feet) that the box can have? (By surface area we mean the sum of the areas of the 5 faces of the box.)

12 16 10 14 8