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theoremTheorem corollary[theorem]Corollary definitionDefinition

Brief Article The Author document

Math 105 Final Exam May 6, 1998 p.6 I have observed the honor code \_\_\_\_\_

Suppose that when a busy restaurant charges \$9 for their octopus appetizer, an average of 50 people order the dish each night. When they raise the price of the appetizer to \$12, 41 people order it each night. Assuming that the demand  $q$  is a linear function of the price  $p$ , find an equation for  $q$  in terms of  $p$ .

$$q = -3p + 77 \quad q = -3p^2 + 5p \quad q = 50p - 9 \quad q = 3p + 23 \quad q = 3p + 5$$

The same restaurant also serves sea cucumber. The sea cucumber demand curve—that is, the number of people who order sea cucumber when the price of the dish is  $p$  dollars—is given by  $q = 80 - 2p$ . What price should the restaurant charge per dish to maximize revenue?

\$20 \$40 \$10 \$30 \$80

Which of the following is equal to  $\ln(3\sqrt{e})$ ?

$$\frac{1}{2} + \ln 3 \quad (\ln 3)(\sqrt{\ln e}) \quad \frac{1}{2}(\ln 3 + \ln e) \quad \frac{1}{3\sqrt{e}} \quad (\sqrt{e})^3$$

Find the area under the curve  $y = \sqrt{x}$  from  $x = 0$  to  $x = 4$ .

$$\frac{16}{3} \quad \frac{2}{3} \quad 12 \quad \frac{4}{3} + C$$

Suppose that the number of fruit flies in a biology experiment grows exponentially, so that the population  $P$  after  $t$  days is given by the equation  $P = Ae^{kt}$  for some constants  $A$  and  $k$ . If the population triples in 8 days, find the growth constant  $k$ .

$$\frac{\ln 3}{8} \quad \frac{\ln 3}{\ln 2} \quad \frac{\ln 2}{8} \quad \frac{\ln(2A)}{8} \quad \frac{\ln(3A)}{8}$$

Let  $f(x) = x - \ln x$  for  $x > 0$ . Where is  $f(x)$  increasing and where is it decreasing?

decreasing on  $(0, 1)$ , increasing on  $(1, \infty)$  increasing on  $(0, 1)$ , decreasing on  $(1, \infty)$  increasing on  $(0, 1)$  and  $(e, \infty)$ , decreasing on  $(1, e)$  decreasing on  $(0, 1)$  and  $(e, \infty)$ , increasing on  $(1, e)$  increasing for all  $x > 0$

Find  $\frac{d}{dx} \left( \frac{x^3}{x^2 + 1} \right)$ .

$$\frac{x^4 + 3x^2}{(x^2 + 1)^2} \quad \frac{3x^2}{2x} \quad \frac{3x^2}{(x^2 + 1)^2} \quad \frac{x^3}{(x^2 + 1)^2} \quad \frac{x^4 + 4x^2}{x^2 + 1}$$

Which of these is equal to  $\int (2e^{2x} + 3x - \frac{3}{x^2}) dx$ ?

$e^{2x} + \frac{3}{2}x^2 + \frac{3}{x} + C$   $4e^{2x} + 6x^2 + \frac{3}{x} + C$   $4e^{2x} + 3x^2 + \frac{1}{x^3} + C$   $2e^{2x} + 3x^2 - \frac{3}{x} + C$   $e^{2x} + \frac{3}{2}x^2 - 3\ln(x^2) + C$   
1:bcead 2:debca

The formula for the area of a circle with radius  $r$  is  $A = \pi r^2$ . Suppose that when the radius is 6 feet, the area is increasing at a rate of 10 square feet per second. How fast is the radius increasing when the radius is 6 feet?

$\frac{5}{6\pi}$  ft./sec.  $120\pi$  ft./sec. 3 ft./sec.  $\sqrt{\frac{10}{\pi}}$  ft./sec.  $12\pi$  ft./sec. 1:dbcea 2:acdbe

Find the value of  $x$  where the tangent line to the graph of  $y = x^2 - 3x + 6$  is parallel to the line  $x + y = 7$ .

$x = 1$   $x = -1$   $x = 0$   $x = 7/3$   $x = -3/7$  1:dbcae 2:eachd

Find all vertical asymptotes, if any, for the function  $\frac{\ln x}{x - 2}$ .

$x = 0$  and  $x = 2$   $y = 0$  and  $y = 2$   $x = 0$   $x = 2$  no vertical asymptotes 1:cdabe 2:eachb

The natural log of 2 is approximately 0.693. Using linear approximation, what is the approximate value of  $\ln(2.2)$ ? (**Warning:** the answer your calculator gives may not be the correct answer to this problem.) 0.793 0.788 0.743 0.801 0.593 1:dabce 2:ecbad

Here is the graph of a function  $f(x)$ . Where does  $f(x)$  have critical points and inflection points?

3in by 1.75in (graph scaled 450)

critical points at  $x = -1$  and  $x = 2$ , inflection points at  $x = 0$  and  $x = 4$ . critical points at  $x = 0$  and  $x = 4$ , inflection points at  $x = -1$  and  $x = -2$ . no critical points, no inflection points critical points at  $x = -1$ ,  $x = 0$ ,  $x = 2$ , and  $x = 4$ ; inflection points at  $x = -2$ ,  $x = 10$  critical points at  $x = -2$  and  $x = 0$ , inflection points at  $x = -1$  and  $x = 2$ . 1:abcde 2:cedba

Find  $\frac{d}{dx}(10^x)$ .

$10^x \ln 10$   $10^x$   $x10^{x-1}$   $10^x \ln x$   $x10^{x-1} \ln 10$  1:bceda 2:abcde

Find all horizontal asymptotes, if any, for the function  $f(x) = \frac{2x^2 + 3x - 7}{x^2 - 1}$ .  $y = 2$   $y = 7$   $y = 0$   $x = 1$  and  $x = -1$  no horizontal asymptote 1:abcde 2:ecadb

Which of the following limits equals the derivative of  $e^{3x}$ ?

$\lim_{h \rightarrow 0} \frac{e^{3x+3h} - e^{3x}}{h}$   $\lim_{h \rightarrow 0} \frac{e^{3x-3h} - e^{3x}}{h}$   $\lim_{h \rightarrow 0} \frac{e^{3x+3h} + e^{3x}}{h}$   $\lim_{h \rightarrow 0} \frac{e^{3x+h} - e^{3x}}{h}$   $\lim_{h \rightarrow 0} \frac{e^{3x+3h} - e^{3x}}{3h}$  1:daebc 2:abcde

Let  $f(x) = \frac{x^3}{3} - 2x^2 + 3x - 1$ . Where does  $f(x)$  have its absolute minimum and maximum on the interval

$[-3, 3]$ ?

minimum at  $x = -3$ ; maximum at  $x = 1$  minimum at  $x = 3$ ; maximum at  $x = 1$  minimum at  $x = -3$ ;  
maximum at  $x = 3$  minimum at  $x = -3$ ; maximum at  $x = -1$  no minimum; maximum at  $x = -1$  1:cbaed  
2:cdeba

Suppose \$2,000 is deposited in an account paying 5% annual interest compounded continuously. Assuming no further deposits or withdrawals are made, how many years will it take for the balance to reach \$6,000?

$$\frac{\ln 3}{0.05} \frac{3}{\ln(0.05)} \frac{\ln 3}{5} e^{1.5} e^{0.05 \ln 3} \quad 1:\text{badce} \quad 2:\text{cdaeb}$$

Suppose \$500 is deposited in an account that pays 6% annual interest, compounded 4 times per year. Assuming no further deposits or withdrawals are made, how much will be in the account at the end of 3 years?

$$500(1.015)^{12} \quad 500(1.18)^4 \quad 500(1.06)^{12} \quad 500(1.02)^4 \quad 500(1.24)^3 \quad 1:\text{edcba} \quad 2:\text{bacde}$$

Which of the following is the derivative of  $\frac{e^{3x^2}}{x-2}$ ?

$$\frac{(6x^2 - 12x - 1)e^{3x^2}}{(x-2)^2} \quad \frac{(x-3)e^{3x^2}}{(x-2)^2} \quad \frac{(6x^2 - 12x - 1)e^{3x^2}}{x-2} \quad \frac{(13-3x)e^{3x^2}}{(x-2)^2} \quad \frac{(6x^2 - 12x - 2)e^{3x^2}}{(x-2)^2} \quad 1:\text{badec} \quad 2:\text{deabc}$$

The function  $f(x) = (x-1)^4$  has only one critical point, at  $x = 1$ . Apply the first and second derivative tests for a relative extremum at that critical point. Which of the following statements is true?

The first derivative test says that  $f(x)$  has a relative minimum at  $x = 1$ ; the second derivative test is inconclusive. The first derivative test is inconclusive; the second derivative test says that  $f(x)$  has a relative minimum at  $x = 1$ . The first derivative test says that  $f(x)$  has a relative minimum at  $x = 1$ ; the second derivative test says that  $f(x)$  has a relative maximum at  $x = 1$ . Both tests say that  $x = 1$  is a relative minimum. The first derivative test says that  $f(x)$  has a relative minimum at  $x = 1$ ; the second derivative test says that at  $x = 1$ ,  $f(x)$  has neither a relative maximum nor a relative minimum. 1:cbade 2:bcdae

If  $F(x)$  is an antiderivative of  $\frac{1}{\sqrt{x}}$  and  $F(1) = 0$ , what is  $F(9)$ ?

$$4 \quad 6 \quad 13/27 \quad 1/54 \quad 8 \quad 1:\text{dbcae} \quad 2:\text{acedb}$$

Where does the function  $f(x) = x^5 - 5x^4 + x - 2$  have inflection points, if any?

Only at  $x = 3$  Only at  $x = 0$  Only at  $x = -1$  At  $x = 0$  and  $x = 3$  At  $x = 0$  and  $x = -1$  1:debca 2:bacde

A certain function  $f(x)$  has first and second derivatives as follows:

$$f'(x) = \frac{-2x}{(x^2 - 1)^2} \quad \text{and} \quad f''(x) = \frac{6x^2 + 2}{(x^2 - 1)^3}.$$

Which of the following most closely resembles the graph of  $f(x)$ ? (Note: the dashed lines are not part of the graph. They are only there to indicate asymptotes.)

Use the letter to the lower left of the picture.

1.75in by 1.75in (graph1 scaled 450) 1.75in by 1.75in (graph2 scaled 450) 1.75in by 1.75in (graph3 scaled 450) 1.75in by 1.75in (graph4 scaled 450) 1.75in by 1.75in (graph5 scaled 450) 1:abcde 2:dbeca

If the **derivative**  $f'(x)$  is decreasing for  $x < a$  and increasing for  $x > a$ , which of the following must be true about the **function**  $f(x)$ ?

It is concave down for  $x < a$  and concave up for  $x > a$ . It is concave up for  $x < a$  and concave down for  $x > a$ . It is decreasing for  $x < a$  and increasing for  $x > a$ . It is increasing for  $x < a$  and decreasing for  $x > a$ . None of the other statements is necessarily true. 1:cdbae 2:badce

Suppose  $f(x)$  and  $g(x)$  are functions satisfying

$$f(0) = 6, \quad g(0) = 3, \quad f'(0) = 5, \quad g'(0) = -1.$$

Let  $h(x) = \ln(f(x) + g(x))$ . What is  $h'(0)$ ?

4/9 1/4 1/9 4 2 1:abcde 2:edcba

An object whose initial temperature is  $5^\circ\text{C}$  is placed in a large room whose temperature is kept at a steady  $20^\circ\text{C}$ . According to Newton's law of cooling, the temperature is given by the formula  $H = 20 + Ae^{kt}$ , where  $H$  is the temperature (in degrees Celsius) and  $t$  is the time in hours. At the end of 1 hour the object has warmed to  $10^\circ\text{C}$ . Find the constants  $A$  and  $k$ .

$A = -15, k = \ln(2/3)$   $A = -15, k = \ln(3/2)$   $A = 5, k = -\ln 2$   $A = -15, k = \ln 2$   $A = 5, k = -0.15$   
1:cabde 2:cdbae

If you apply linear approximation to the function  $f(x) = x^{-1/2}$  to approximate  $(4 + h)^{-1/2}$ , where  $h$  is a very small number, what do you get?

$$\frac{1}{2} - \frac{h}{16} \quad \frac{1}{2} + \frac{h}{4} \quad \frac{1}{2+h} \quad \left(2 + \frac{h}{16}\right)^{-3/2} \quad \left(2 + \sqrt{h}\right)^{-1} \quad 1:dbcae \quad 2:adbec$$

Where are the absolute maximum and minimum, if any, of the function  $f(x) = xe^{-x}$ ?

absolute maximum at  $x = 1$ , no absolute minimum no absolute maximum, absolute minimum at  $x = 1$   
absolute maximum at  $x = 1$ , absolute minimum at  $x = 0$  absolute maximum at  $x = 0$ , absolute minimum at  $x = 1$  no absolute maximum or minimum 1:cdeba 2:bcade

Find the equation of the line tangent to the curve  $y = 2x \ln x$  at the point  $(1, 0)$ .

$y = 2x - 2$   $y = 0$   $y = 4x - 4$   $y = 2 \ln x$   $y = -2x$  1:cbade 2:dceab