

Part I. Multiple Choice

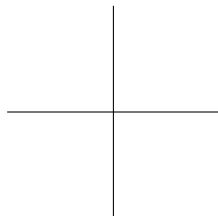
1. Last month Mary rented a car twice from the Cars-R-Us Car Rental Company, which charges a certain amount just for renting the car, plus a certain amount per mile. The first time, she drove the car 200 miles and was charged \$120. The second time, she drove the car 50 miles and was charged \$45. How much does Cars-R-Us charge per mile? **Ans.** 0.50. Cost is a linear function of miles driven, and the slope is the charge per mile.
2. Find the y -intercept of the line $x + 2y = 3$. **Ans.** $3/2$. Simply solve the equation for y .
3. If $f(x) = \frac{(x-1)^2}{(x+1)^2}$, find $f(a+1)$. **Ans.** $\frac{a^2}{(a+2)^2}$. Replace x by $a+1$ wherever it appears.
4. A rock is thrown from the top of a 48-foot tower straight up into the air at time $t = 0$ with an initial velocity of 32 feet per second. Its height h at time t is given by the formula $h = -16t^2 + 32t + 48$. At what time does the rock hit the ground?
Ans. $t = 3$. Solve $h = 0$
5. It is determined that a certain company earns a profit of $200x - 2x^2$ (in dollars) per day for producing x units of their product. What is the maximum possible profit that they can earn in one day? **Ans.** \$5,000. Completing the square expresses the profit as $-2(x-50)^2 + 5000$, which is maximum when $x = 50$ with a maximum value of 5000.
6. What is the natural domain of the function $f(x) = \frac{5x}{(x^2+1)(\sqrt{x-2})}$? **Ans.** $x > 2$.
To avoid having the square root of a negative quantity, we must have $x \geq 2$, and to avoid a zero in the denominator we must have $x \neq 2$. There are no other restrictions.
7. Suppose the function f is given by the multi-line definition

$$f(x) = \begin{cases} x^2, & x \leq 1; \\ x - 1, & 1 < x < 3; \\ -\frac{1}{3}x + 3, & x \geq 3. \end{cases}$$

Where does f have discontinuities? **Ans.** At $x = 1$ only. There are only two points we have to worry about, one with $x = 1$ and the other with $x = 3$. At the former, the left-hand branch approaches height 1 and the right-hand branch approaches height 2, so there is a discontinuity. At the latter, both branches approach height 2, so there is no discontinuity.

8. Which of the following could be the graph of $y = b^x$ for some $b > 1$?

Ans.



It's the only graph that lies entirely above the x axis, rises to the right, and falls asymptotically to the x axis to the left.

9. Jack invests \$5000 in an account that earns an annual rate of 7%, compounded monthly. If he makes no further deposits or withdrawals, what will be the value of the account after one year (in dollars)? **Ans.** $5000 \left(1 + \frac{.07}{12}\right)^{12}$. That is the compound interest formula with $r = 0.07$, $n = 12$, $A = 5000$, and $t = 1$

Part II. Partial Credit

10. A widget company finds that in order to make x widgets in a given month, their cost function (in dollars) is $C(x) = 1000 + 3x$. Suppose they charge \$5.00 per widget.

(a) Express their monthly profit function as a function of x . **Ans.** $2x - 1000$, because $R(x) = 5x$ and $P(x) = R(x) - C(x)$.

(b) How many widgets should they produce in order to break even? **Ans.** 500. Solve $P(x) = 0$.

11. (a) What is the slope of the line $2x + 3y = 4$? **Ans.** $-2/3$. Solve the equation for y .

(b) Find the equation of the line through the point $(-1, -2)$ and parallel to the line $2x + 3y = 4$. **Ans.** $y = -(2/3)x - (8/3)$ or $2x + 3y = -8$. Either write the equation of the line through $(-1, -2)$ with slope $-2/3$ or write $2x + 3y = C$ and plug in $(-1, -2)$ to find C .

12. The following is a table of values for some function.

x	0	2	4	6	8	10
y	18	15	12	9	6	3

If the function is linear, write it explicitly in the form $y = mx + b$. If it is not linear, explain clearly why it is not. **Ans.** $y = -(3/2)x + 18$. The change in y divided by the change in x is constant, and that is the slope.

13. (a) Complete the square of the quadratic equation $y = 2x^2 + 4x + 6$.

Ans. $y = 2(x + 1)^2 + 4$.

- (b) Does the graph of the equation in (a) open upward or downward? **Ans.** Upward.
The coefficient of the second-degree term is positive.

14. The following table of values does *not* correspond to a linear function.

x	0	2	4	6	8	10
y	18	16	16	14	13	10

Using the method described in class, find **just** the slope of the linear function that approximates the data. **Ans.** $-4/5$. The average y -change is $-8/5$, and the x -change is 2.

15. Consider the graph of the function

$$f(x) = \frac{x}{(x^2 - 9)}.$$

- (a) Find its *horizontal* asymptote, if there is one. **Ans.** $y = 0$.
(b) Find its *vertical* asymptote(s), if there are any. **Ans.** $x = 3$ and $x = -3$.

16. The line $x = -2$ is a vertical asymptote for the function

$$g(x) = \frac{x^2 + 6}{x + 2}.$$

Determine the behavior of the graph (i.e. positive or negative) as it approaches the asymptote from either side. **Ans.** From the right the graph is positive. From the left the graph is negative.

17. Let $f(x) = x^{(-\frac{3}{2})}$.

- (a) What is the domain of f ? **Ans.** $x > 0$.
(b) What is $f(9)$? **Ans.** $1/27$

18. The following tables give values from three functions. One function is linear, one is exponential, and one is neither.

I.

x	0	1	2	3	4	5
$f(x)$	6	12	24	48	96	192

II.

x	0	1	2	3	4	5
$g(x)$	8	20	50	100	160	220

III.

x	0	1	2	3	4	5
$h(x)$	5	7.3	9.6	11.9	14.2	16.5

(a) Determine which function is linear, which is exponential, and which is neither.

Ans. Table I. is exponential. Table II. is neither. Table III. is linear.

(b) Find a formula $y = Ab^x$ for the exponential function. **Ans.** $y = 6 \cdot 2^x$. The ratio of each pair of successive y values is 2, which equals b .

19. (5 points) According to a recent *Headline News* broadcast, the cost of education at a state school may be as high as \$100,000 eighteen years from now.

Suppose the parents of a newborn want to put some money away today for their child's education. If their savings account pays simple interest figured annually at a rate of 5%, how much should they contribute now in order to have \$100,000 in 18 years?

Ans. \$41,552.07. The compound interest formula gives $100,000 = A(1.05)^{18}$, so that $A = 100,000/(1.05)^{18} \approx 41,552.065$.