Math 105 **Answers to test 2**

Part I. Multiple Choice

1. Which of the following expressions is equivalent to $e^{\ln t+\ln(1/t)}$? **Ans.** 1.
Because $\ln t+\ln(1/t)=\ln\left(t\cdot\frac{1}{t}\right)=\ln 1=0$.

2. Given a function $y=f(x)$, we can calculate the derivative by using the formula

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\quad (\textbf{Ans.})$$

3. If you deposit \$500 in an account paying 4% interest compounded continuously, how much will be in the account after 20 years? **Ans.** $500e^{0.8}$. From the formula for continuously compounded interest, $y=Ae^{rt}$, with $A=500$, $r=0.04$ and $t=20$

4. Which of the following limits would you obtain when using the definition of the derivative to compute $f'(x)$ where $f(x)=x^2-x$? **Ans.** $\lim_{h\to 0}\dfrac{2xh+h^2-h}{h}$. Apply the basic definition (as given in question 2 above) and simplify.

5. What is the derivative $f'(x)$ of the function $f(x)=3x^4-\frac{4}{\sqrt{x}}$? **Ans.** $12x^3+2x^{-3/2}$. Rewrite $f(x)=3x^4-4x^{-1/2}$ and use the power rule with the addition and coefficient rules.

6. A company finds that if it produces and sells x boxes of assorted chocolates per week, its profit (in dollars) is $-1000+15x+0.02x^2$. What is its marginal profit (in dollars) at a production level of 100 boxes per week? **Ans.** \$19.00. The marginal profit function is $P'(x)=15+0.04x$, so $P'(100)=19$.

7. If $y=e^{2t}-2e^t$, which of the following functions is equal to $\frac{dy}{dt}$? **Ans.** $2e^{2t}-2e^t$. Use the formula $\frac{d}{dt}e^{ct}=ce^{ct}$ with the addition and coefficient formulas.

8. If $g(x)=3x^4-2x^3+5x-\ln x$, which of the following functions is equal to $g''(x)$?
Ans. $36x^2-12x+\dfrac{1}{x^2}$, because $g'(x)=12x^3-6x^2+5-\dfrac{1}{x}$.

9. What is the point on the curve $y=\ln x$ where the tangent line is parallel to the line $3y=2x+1$? **Ans.** $(\frac{3}{2},\ln\frac{3}{2})$. Solve the equation $\frac{1}{x}=\frac{2}{3}$.

10. With an inflation rate of 2% per annum, prices are described by the formula $P=P_0(1.02)^t$ where P is the price in dollars after t years and P_0 is the price when $t=0$. How fast are prices rising in dollars per year when $t=3$? **Ans.** $P_0(1.02)^3\ln(1.02)$. Use the formula $\frac{d}{dt}b^t=b^t\ln b$ with $b=1.02$, then plug in $t=3$.

Part II. Partial Credit

11. Find all values of x which satisfy the equation $4+\ln(x^3)=10$. **Ans.** $x=e^2$ is the only solution. Rewrite as $\ln(x^3)=6$, so that $x^3=e^6$, and $x=(e^6)^{1/3}=e^2$.

12. You are considering investing some money with Investments 'R' Us. When you do some research, you find that they plan to invest your money in bonds paying 7% interest compounded continuously. How long will it take Investments 'R' Us to triple your investment? **Ans.** $(\ln 3)/0.07 \approx 15.7$ years. With A being the initial investment and t the time in years, you have to solve the equation $3A = Ae^{0.07t}$.

13. Tritium, a synthetic form of hydrogen, satisfies the exponential decay law, $y = Ae^{-kt}$, where t is the time (in years) and y is the amount (in grams) at time t. If an absent-minded scientist receives a shipment of 200 grams of tritium and finds that only 50 grams of it are left when he remembers the sample 25 years later, find a function that gives the amount of tritium remaining in the sample t years after the scientist received the shipment.
Ans. $200e^{-t(\ln 4)/25}$ or $200e^{t(\ln(1/4))/25}$ or (approximate) $200e^{-0.05545t}$.
First solve the equation $50 = 200e^{-25k}$ for k, then substitute that value of k into the decay equation, also setting $A = 200$.

14. Find the equation of the tangent line to the curve $y = 2x^3 - 2x$ at the point $(1, 0)$.
Ans. $y = 4x - 4$. First compute the derivative, $y' = 6x^2 - 2$, then plug in $x = 1$ to find the slope at $(1, 0)$. Then write the equation of the line through $(1, 0)$ with that slope.

15. A toy rocket launched straight up from the top of a building has a height s in feet above ground given by the formula $s = 96 + 80t - 16t^2$, where t denotes the time in seconds after launch. (a) When is the velocity of the rocket zero? **Ans.** At $t = 5/2$ sec. You have to solve the equation $dy/dt = 0$. (b) What is the velocity of the rocket when it strikes the ground? **Ans.** -112 ft. per sec. First solve the equation $96 + 80t - 16t^2 = 0$ to find when the rocket strikes the ground. The only positive solution is $t = 6$. Then plug that into the derivative $y' = 80 - 32t$.

16. Let $P(x)$ be the profit made by a company selling x midsize cars per year. Suppose that $P(100) = 60,000$ and $P'(100) = 1,000$. Estimate the profit from the sale of 102 cars per year. **Ans.** \$62,000. Use the linear approximation formula $P(102) \approx P(100) + P'(100) \cdot 2$.

17. Calculate the derivative of the function $y = \ln \sqrt[3]{x}$. **Ans.** $y' = \frac{1}{3x}$. Rewrite $y = \frac{1}{3} \ln x$, so that $y' = \frac{1}{3} \frac{d}{dx} \ln x = \frac{1}{3x}$

18. The distance s in feet travelled by a car during the first ten seconds after it starts from rest is given by the formula $s = \frac{5}{2}t^3 - \frac{4}{7}t^{7/2}$, where t is the time in seconds. (a) What is the average velocity of the car in the first second. **Ans.** 27/14 ft. per sec. Since $s(1) = 27/14$ and $s(0) = 0$, the distance travelled is 27/14 ft. The time elaped is 1 sec. (b) What is the acceleration of the car after 4 seconds? **Ans.** 20 ft. per sec^2. Compute s'' and plug in $t = 4$.