Part I: Multiple Choice Questions *(5 points each)*

1. Which of the following expressions is equivalent to $e^{\ln t + \ln(1/t)}$?

$$(a) \quad 1 \qquad (b) \quad t\ln\frac{1}{t} \qquad (c) \quad 0 \qquad (d) \quad t+\frac{1}{t} \qquad (e) \quad t+\ln(\frac{1}{t})$$

2. Given a function $y = f(x)$, we can calculate the derivative by using the formula

$$(a) \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) + f(x)}{h} \qquad\qquad (b) \quad f'(x) = \lim_{h \to 0} \frac{f(x) - f(h)}{h}$$

$$(c) \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \qquad\qquad (d) \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{x - h}$$

$$(e) \quad f'(x) = \lim_{h \to 0} \frac{f(h) - f(x)}{h - x}$$

3. If you deposit $500 in an account paying 4% interest compounded continuously, how much will be in the account after 20 years?

$$(a) \quad 4e^{20} \qquad (b) \quad 500e^{80} \qquad (c) \quad 500(1.04)^{20} \qquad (d) \quad 4e^{10000} \qquad (e) \quad 500e^{0.8}$$

4. Which of the following limits would you obtain when using the definition of the derivative to compute $f'(x)$ where $f(x) = x^2 - x$?

(a) $\displaystyle\lim_{h\to 0} \frac{2xh + h^2 - h - xh^2}{2}$

(b) $\displaystyle\lim_{h\to 0} \frac{x^2h - xh - h^2 + h}{h}$

(c) $\displaystyle\lim_{h\to 0} \frac{2xh + h^2 - h}{h}$

(d) $\displaystyle\lim_{h\to 0} \frac{h^2 - h - x^2 + x}{h - x}$

(e) $\displaystyle\lim_{h\to 0} \frac{2xh + h^2 - h}{x - h}$

5. What is the derivative $f'(x)$ of the function $f(x) = 3x^4 - \frac{4}{\sqrt{x}}$?

(a) $12x^3 - 2x^{-3/2}$

(b) $3x^3 - 4x^{-3/2}$

(c) $12x^4 - 2x^{-1/2}$

(d) $3x^3 + 4x^{-3/2}$

(e) $12x^3 + 2x^{-3/2}$

6. A company finds that if it produces and sells x boxes of assorted chocolates per week, its profit (in dollars) is $-1000 + 15x + 0.02x^2$. What is its marginal profit (in dollars) at a production level of 100 boxes per week?

(a) $15.02

(b) $19.00

(c) $15.04

(d) $17.00

(e) $15.20

7. If $y = e^{2t} - 2e^t$, which of the following functions is equal to $\frac{dy}{dt}$?

   (a) $2te^t - 2$ (b) $2te^{2t-1} - 2te^{t-1}$ (c) $e^{2t} - 2e^t$

   (d) $2e^t - 2$ (e) $2e^{2t} - 2e^t$

8. If $g(x) = 3x^4 - 2x^3 + 5x - \ln x$, which of the following functions is equal to $g''(x)$?

   (a) $36x^4 - 12x^3 + \frac{1}{x^2}$ (b) $3x^2 - 2x - \frac{1}{x^2}$ (c) $36x^2 - 12x + \frac{1}{x^2}$

   (d) $36x^4 - 12x^3 - \frac{1}{x^2}$ (e) $3x^2 - 2x + \frac{1}{x^2}$

9. What is the point on the curve $y = \ln x$ where the tangent line is parallel to the line $3y = 2x + 1$?

   (a) $(e^{3/2}, \frac{3}{2})$ (b) $(\frac{2}{3}, \ln \frac{2}{3})$ (c) $(e^{2/3}, \frac{2}{3})$

   (d) $(\frac{3}{2}, \ln \frac{3}{2})$ (e) $(e^{-3/2}, -3/2)$

10. With an inflation rate of 2% per annum, prices are described by the formula $P = P_0(1.02)^t$ where P is the price in dollars after t years and P_0 is the price when $t = 0$. How fast are prices rising in dollars per year when $t = 3$?

   (a) $P_0(1.02)^3$ (b) $P_0(1.02)^3 \ln(1.02)$ (c) $P_0 \ln(1.02)$

   (d) $3P_0(1.02)^2 \ln(1.02)$ (e) $0.02P_0$

Part II: Partial Credit Questions

Show all work and put your final answer in the space provided. You will receive no credit if the answer is not in the space provided and no partial credit for a wrong answer if you do not show your work.

Answers may be approximated by decimals if desired, but full points will also be awarded for correct answers left in exact form in terms of logarithms or exponentials.

11. *(5 points)* Find all values of x which satisfy the equation $4 + \ln(x^3) = 10$.

Answer:_____

12. *(5 points)* You are considering investing some money with Investments 'R' Us. When you do some research, you find that they plan to invest your money in bonds paying 7% interest compounded continuously. How long will it take Investments 'R' Us to triple your investment?

Answer:_____

13. *(6 points)* Tritium, a synthetic form of hydrogen, satisfies the exponential decay law, $y = Ae^{-kt}$, where t is the time (in years) and y is the amount (in grams) at time t. If an absent-minded scientist receives a shipment of 200 grams of tritium and finds that only 50 grams of it are left when he remembers the sample 25 years later, find a function that gives the amount of tritium remaining in the sample t years after the scientist received the shipment.

Answer:_____

14. *(6 points)* Find the equation of the tangent line to the curve $y = 2x^3 - 2x$ at the point $(1, 0)$.

Answer:_____

15. A toy rocket launched straight up from the top of a building has a height s in feet above ground given by the formula $s = 96 + 80t - 16t^2$, where t denotes the time in seconds after launch.

(a) *(4 points)* When is the velocity of the rocket zero?

Answer:_____

(b) *(4 points)* What is the velocity of the rocket when it strikes the ground?

Answer:_____

16. *(5 points)* Let $P(x)$ be the profit made by a company selling x midsize cars per year. Suppose that $P(100) = 60,000$ and $P'(100) = 1,000$. Estimate the profit from the sale of 102 cars per year.

Answer:_____

17. *(5 points)* Calculate the derivative of the function $y = \ln \sqrt[3]{x}$.

Answer:_____

18. The distance s in feet travelled by a car during the first ten seconds after it starts from rest is given by the formula $s = \frac{5}{2}t^3 - \frac{4}{7}t^{7/2}$, where t is the time in seconds.

(a) *(4 points)* What is the average velocity of the car in the first second.

Answer:_____

(b) *(6 points)* What is the acceleration of the car after 4 seconds?

Answer:_____