Part I: Multiple Choice Questions (5 *points each*)

1. $\dfrac{d}{dx}(x^3 e^{4x}) =$

(a) $12x^2 e^{4x}$ (b) $3x^2 e^{4x} + 4x^4 e^{4x-1}$ (c) $x^3 e^{4x} + 12x^2 e^{4x}$

(d) $3x^2 e^{4x} + 4x^3 e^{4x}$ (e) $4x^3 e^{4x-1}$

2. Suppose $f(x)$ is a function which satisfies $f'(3) = 0$, $f'(5) = 0$, $f''(3) = -4$, and $f''(5) = 5$. Which of the following statements is true?

(a) $f(x)$ has a relative minimum at $x = 3$ and at $x = 5$.
(b) $f(x)$ has a relative minimum at $x = 3$ and a relative maximum at $x = 5$.
(c) $f(x)$ has a relative maximum at $x = 3$ and at $x = 5$.
(d) $f(x)$ has a relative maximum at $x = 3$ and a relative minimum at $x = 5$.
(e) None of the above is true.

3. Let $y = \sqrt{\ln x}$. Then $y' =$

(a) $\dfrac{1}{2x}$ (b) $\dfrac{1}{2x\sqrt{\ln x}}$ (c) $\dfrac{1}{2\sqrt{x}}$ (d) $\dfrac{1}{2\sqrt{\ln x}}$ (e) $\dfrac{x}{2\sqrt{\ln x}}$

Questions 4 - 5 involve the following table of values

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	2	-1	$1/2$
1	0	3	$1/2$	0
2	$1/3$	$5/2$	2	$-1/3$

4. Let $h(x) = \dfrac{f(x)}{g(x)}$. Then $h'(0) =$

(a) 2 (b) $-3/8$ (c) $5/8$ (d) -2 (e) $-5/8$

5. Let $k(x) = f(g(x))$. Then $k'(0) =$

(a) $1/6$ (b) $-1/2$ (c) 1 (d) 4 (e)$-2/3$

6. Let $f(x)$ be a function with $f'(2) = 0$. Which of the following **must** be true about f?

(a) At $x = 2$, f changes from increasing to decreasing or from decreasing to increasing.
(b) The tangent line to the graph of f at $(2, f(2))$ is the line $x = 2$.
(c) f has a local maximum or a local minimum at $(2, f(2))$.
(d) $(2, f(2))$ is an inflection point.
(e) The tangent line to the graph of f at $(2, f(2))$ is horizontal.

7. Suppose that $f(x)$ is increasing and concave up everywhere. Further, $f(2) = 2$ and the tangent line at the point $(2, 2)$ is the line $y = x$. Which of the following can you conclude?

(a) $f(5) > 5$ (b) $f(5) = 2$ (c) $f(5) < 5$ (d) $f(5) = 5$ (e) none of the preceding

8. The graph of a function $f(x)$ is sketched below. Which of the following statements is true?

(a) $f(x)$ is increasing and $f'(x)$ is decreasing.
(b) Both $f(x)$ and $f'(x)$ are increasing.
(c) $f(x)$ is decreasing and $f'(x)$ is increasing.
(d) Both $f(x)$ and $f'(x)$ are decreasing.
(e) None of the above is true.

9. Suppose that $f(x)$ is a function with $f''(x) > 0$ on the interval $(2, \infty)$ and $f''(x) < 0$ on the interval $(-\infty, 2)$. Which of the following could be the graph of $f(x)$? (Use the letter to the upper left of each graph.)

(a) (b) (c)

(d) (e)

10. Suppose that $f(x)$ is a function whose **derivative** is $f'(x) = x \ln x$. Identify the x-value(s) of all inflection point(s) of $f(x)$.

(a) e (b) $0, 1$ (c) 0, e (d) $1/e$ (e) $f(x)$ has no inflection points

Part II: Partial Credit Questions

Show all work and put your final answer in the space provided. You will receive no credit if the answer is not in the space provided and no partial credit for a wrong answer if you do not show your work.
Answers may be approximated by decimals if desired, but full points will be also be awarded for correct answers left in exact form in terms of logarithms or exponentials.

11. (7 *points*) A pot of boiling water (100° Celsius) is left to cool in a room whose temperature is 20° Celsius so that after t minutes, the temperature of the water is $H = 20 + Ae^{kt}$ for some constants A and k. After 30 minutes, the water is 60°. At what **rate** (with respect to time) is the temperature of the water changing after 90 minutes?

Ans. ______________________________

12. (5 *points*) The half-life of carbon-14 is 5730 years, so carbon decays according to the equation $y = Ae^{-kt}$, where $k = .000121$. An ancient document written on papyrus is found to have 37.5 % of the carbon-14 that a freshly cut sample has. How old is the document?

Ans. ______________________________

13. (6 *points*) Find the the equation of the line tangent to $y = \dfrac{e^{x^2+2x}}{x^3+1}$ at $(0, 1)$.

Ans. ______________________________

14. (6 *points*) The period P in seconds of a simple pendulum is completely determined by its length L according to the formula $P = \dfrac{2\pi}{\sqrt{10}}\sqrt{L}$, where $\pi \approx 3.14$. Therefore, if the length of a pendulum increases, its period will increase. (That is why clocks with metal pendulums tend to run slow in the summertime.) Find the rate (with respect to time) at which a simple pendulum's period is increasing when the pendulum is 4m long and its length is increasing at a rate of .001m/day.

Ans. ______________________________

15. (6 *points*) The demand q for Widgets is a function of the price p, i.e. $q = f(p)$. When the price $p = \$10$, we have $q = f(10) = 100$ and $f'(10) = -4$.

(a) The revenue R is determined by $R = pq$. Find $\dfrac{dR}{dp}$ when $p = 10$.

Ans. ____________________

(b) The cost of producing q Widgets is $C(q) = 100q + 5000$. Find $\dfrac{dC}{dp}$ when $p = 10$.

Ans. ____________________

(c) If P represents profit, find $\dfrac{dP}{dp}$ when $p = 10$.

Ans. ____________________

16. (6 *points*) Consider the function $f(x) = x - 3x^{\frac{2}{3}}$.

(a) Identify the intervals on which $f(x)$ is increasing and those on which $f(x)$ is decreasing.

Ans. ______________________

(b) Identify the x-value(s) of all relative extreme point(s) of $f(x)$ and specify whether each is a maximum or a minimum.

Ans. ______________________

17. (6 *points*) Consider the function $f(x) = x^4 - 54x^2 + 27x - 17$.

(a) Identify the intervals on which $f(x)$ is concave up and those on which $f(x)$ is concave down.

Ans. ______________________

(b) Identify the x-value(s) of all inflection point(s) of $f(x)$.

Ans. ______________________

18. (8 *points*) Suppose that $f(x)$ is a function which satisfies the following list of properties:

- $f(-3) = -1$, $f(0) = -3$, $f(2) = -5$, and $f(4) = 0$.
- The only solutions to $f'(x) = 0$ are $x = -3$ and $x = 2$.
    - $f'(x) > 0$ on the intervals $(-\infty, -3)$, $(2, \infty)$.
    - $f'(x) < 0$ on the interval $(-3, 2)$.
- The only solution to $f''(x) = 0$ is $x = 0$.
    - $f''(x) > 0$ on $(0, \infty)$.
    - $f''(x) < 0$ on $(-\infty, 0)$.

(a) List all the relative extreme points and inflection points of $f(x)$.

(b) Sketch a possible graph of $f(x)$, labeling the points you listed in part (a).