latest 6.3in8.5in-1cm0.7cm
document = =2.5cm=usual =3 May 6, 1999 **Math 105, Final Exam** minipage[c]6in This exam is worth 150 points. There are 20 multiple choice questions, each worth 5 points, and 5 partial credit questions. You will have 2 hours to complete the exam. Calculators are allowed. Record your answers to the multiple choice problems by placing an $\times$ in the appropriate box. Answers to problems in the partial credit section may be written in terms of $\ln x$ or e^x, and need not be simplified. Remember that you are taking this examination under the honor code. Good Luck.
[-2mm]0mm6mmSign your name-10cm 6cm11=1in =0.8cm =.7cm =0.4cm=1
What is the equation of the line through the points $(2, 3)$ and $(-1, 9)$?
$y = -12x + 7$ $y = -12x + 4$ $y = -2x + 7$ $y = -2x - 3$ $y = -2x - 4$
Let $f(x) = x^4 - 2x^3 + 3x + 5$. What is $f''(x)$?
$7x^2 - 8x$ $12x^2 - 12x + 8$ $4x^3 - 6x^2 + 3$ $12x^2 - 12x$ $4x^3 - 6x^2 + 5$
If you use linear approximation to estimate $1(1.00136)^5$, what answer do you get?
$1 - 5(0.00136)$ $1 + 6(0.00136)$ $1.00136 - 5(0.00136)^{-6}$ $15(0.00136) + 1$ $15(0.00136) + 1.00136$
What is the equation for the slope of the tangent line to the graph of $f(x) = x^3 \ln x$?
$3x$ $x^2(1 + 3\ln x)$ $x^2 \ln x$ $x(x \ln x + 3)$ $3x^2 \ln x$
Which of the following is true about the function

$$f(x) = 2x^2 + 3x - 7x^2 - 1?$$

$f(x)$ has vertical asymptotes at $x = 1$ and $x = -1$ and a horizontal asymptote at $y = 2$. $f(x)$ has vertical asymptotes at $x = 1$ and $x = -1$ and a horizontal asymptote at $y = 0$. $f(x)$ has a vertical asymptote at $x = 1$ and no horizontal asymptotes. $f(x)$ has a vertical asymptote at $x = 1$ and horizontal asymptotes at $y = 2$ and $y = 1$. $f(x)$ has no vertical asymptotes and no horizontal asymptotes.
The volume of a sphere is expanding. The volume of a sphere of radius r is $V = 43\pi r^3$. The rate of expansion of the radius ($drdt$) is 5 in/sec when $r = 2$. What is $dVdt$ when $r = 2$?
16π 323π 3215π 80π 23π
If an investor opens an account with \$10,000 where interest is 4% compounded continuously, how long will it take for the investment to triple?
$-\ln 30.04$ $\ln 30000$$\ln 10000$ $\ln 30.04$ $\ln 0.043$ $-\ln 0.043$
A ball is thrown into the air at a velocity of 96 ft/sec from the top of a 16 foot tall platform. The equation for the height of the object at any time t is then given by $h(t) = -16t^2 + 96t + 16$. When does the ball hit the ground?
$48 + 16\sqrt{10}$ seconds 3 seconds 6 seconds 2 seconds $3 + \sqrt{10}$ seconds
What are the critical points of the function $f(x) = e^{x^3 - 3x}$?
$x = 1$ $x = 0$ $x = 1, x = -1$ and $x = 0$ $x = 1$ and $x = -1$ There are no critical points
If $f(x) = \sqrt{x^2 + 1}$, which of the following limits equals $f'(x)$?
$\lim_{h\to 0} \sqrt{(x+h)^2+1} - \sqrt{x^2+1}h$ $\lim_{h\to 0} \sqrt{x^2+1} + h - \sqrt{x^2+1}h$ $\lim_{h\to 0} \sqrt{x^2+h+1} - \sqrt{x^2+1}h$ $\lim_{h\to 0} \sqrt{x^2+h^2+1} +$
$\lim_{h\to 0} \sqrt{(x+h)^2+1} + \sqrt{x^2+1}h$
Which of the following is an equivalent expression for $\ln(3e^{2x})$?
$2x \ln 3$ $\ln 3 + 2x$ $6x$ $2x + 3$ $3 \ln 2x$
Suppose $f(x)$ is a function which satisfies the following:
• The only critical points are $x = -1$ and $x = 2$.
• $f'(-3) = 2, f'(0) = -1, f'(3) = -2$.
Which of the following must be true?

$f(x)$ has a relative maximum at $x = -1$ $f(x)$ has a relative minimum at $x = -3$ $f(x)$ is increasing on $(-\infty, 0)$ $f(x)$ is concave up on $(-1, 2)$ $f(x)$ has a relative minimum at $x = 2$

A clock company finds that it can sell each clock for \$20 apiece. If the cost of manufacturing x clocks is $C(x) = 10x + 1300$, what is the company's $\underline{profit}function$?

$P(x) = 30x + 1300$ $P(x) = 200x + 26000$ $P(x) = 10x - 1300$ $P(x) = 10x + 1280$ $P(x) = -10x + 1300$

Compute the derivative of $e^{3x}x^2$.

$xe^{3x}(2 - 3x)x^4$ $3e^{3x}2x$ $3e^{3x}(x^2)2xe^{3x}$ $2xe^{3x}3x^2e^{3x}$ $xe^{3x}(3x - 2)x^4$

Over the past ten years, a certain stock has yielded an annual interest rate of 8%, compounded $\underline{monthly}.If you bought \$3000 wor$

$3000(1 + 0.0812)^{120}$ $3000(1 + 0.08)^{120}$ $3000(1 + 0.0812)^{10}$ $3000e^{0.08}$ $3000e^{120}$

What is the equation of the tangent line to $f(x) = e^x + 1$ at the point $(0, 2)$?

$y = 2x + 3$ $y = x + 2$ $y = 12x + 2$ $y = 1e(x - 2)$ $y = x + 5$

A hard-boiled egg of 150°F is set to cool in a room of 60°F. If after 1 minute the egg has cooled to 140°F, then what is the equation for the temperature of the egg at any time t? Recall that Newton's Law states that the temperature of the egg at any time t is given by the equation $H(t) = 60 + Ae^{kt}$.

$H(t) = 60 + 90e^{\ln(8/9)t}$ $H(t) = 60 + 90e^{\ln(4/9)t}$ $H(t) = 60 + 90e^{\ln(-1/9)t}$ $H(t) = 60 + 10e^{\ln(8/9)t}$ $H(t) = 60 + 10e^{\ln(-1/9)t}$

Compute the derivative of $\ln(2x) - 3x^2 + 2$.

$12x - 6x$ $1x - 6$ $12 + 1x - x$ $1x - 6x$ $12 + x - 6x$

Let

$$f(x) = \begin{cases} -x^2 + 3, & \text{if } x < 0; \\ -2x + 2, & \text{if } 0 \le x < 2; \\ x^2 - 6, & \text{if } x \ge 2. \end{cases}$$

Where is $f(x)$ discontinuous?

$x = 0$ and $x = 2$ $x = 2$ $x = 0$ $x = 3$ nowhere

If $f(x) = (x^3 - 4x^2 + 6)^{1/2}$, then what is $f'(x)$?

$12(x^3 - 4x^2 + 6)^{-1/2}(3x^2 - 8x)$ $12(x^3 - 4x^2 + 6)^{-1/2}$ $12(3x^2 - 8x)^{-1/2}$ $32x^{1/2} - 4$ $32x^{1/2} - 4 + 12\sqrt{6}$

(10 points)

A photographer knows that she can sell 90 reproductions of one of her prints for \$20 each. For each \$1 increase in price, she will sell three fewer prints. How much should she charge per print in order to maximize revenue?

(10 points)

Where is the function $f(x) = x^4 - 2x^2 + 1$ concave up and where is it concave down?
(8 points)

Consider the graph of the function $g(x)$.

Where is this function increasing, and where is it decreasing?

Does this function have relative maximum or minimum points, and if so where?

Where does the function appear to be concave up and concave down?

What are the horizontal and vertical asymptotes of this function?

(12 points) Suppose that $f(x)$ is a function which satisfies the following list of properties:

- $f(-1) = 3, f(0) = 4, f(1) = 0, f(2) = -2$, and $f(3) = 0$.
- The only solutions to $f'(x) = 0$ are $x = 0$ and $x = 2$.
  - $f'(x) > 0$ on $(-\infty, 0)$ and $(2, \infty)$
  - $f'(x) < 0$ on $(0, 2)$
- The solutions to $f''(x) = 0$ are $x = -1, x = 1$ and $x = 3$.
  - $f''(x) > 0$ on $(-\infty, -1)$ and $(1, 3)$
  - $f''(x) < 0$ on $(-1, 1)$ and $(3, \infty)$
- $f(x)$ has no vertical asymptotes.
- $f(x)$ has a horizontal asymptote at $y = 2$.

Using this information, sketch the graph of $f(x)$.

(10 points)

Consider the function

$$g(x) = x^2 + 3x - 1x - 2.$$

The function $g(x)$ has critical points at $x = 5$ and $x = -1$. Given that the second derivative is

$$g''(x) = 18(x - 2)^3,$$

determine whether the critical points are relative maxima or relative minima.