1. If $f(x) = \dfrac{x}{x-3}$, for which value(s) of $x$ does $f(x)$ have a rel. max or min?

(a) none          (b) $x = -1$ and $x = -3$          (c) $x = 0$          (d) $x = 1$          (e) $x = 3$

2. Suppose $y = e^x$ and $x = t^2$. Find $\dfrac{dy}{dt}$ when $t = 10$.

(a) $20e^{100}$          (b) $e^{100}$          (c) $20$          (d) $200$          (e) $100e^{20}$

3. Let $f(x) = (\ln x)/x$. On which of the following intervals is $f(x)$ increasing?

(a) $(0, e)$          (b) $(1, 2e)$          (c) $(\ln 2, 4)$          (d) $(e, \infty)$          (e) no interval

4. A ball is thrown up into the air so that its height $h(t)$ after $t$ seconds is given by the formula $h(t) = -16t^2 + 128t + 5$. Find the maximum height that the ball reaches.

(a) 261          (b) 256          (c) 266          (d) 278          (e) 237

5. Where does the function $f(x) = e^{-x^2}$ have an absolute maximum?

(a) $x = 0$          (b) $x = -1/\sqrt{2}$          (c) $x = 1/\sqrt{2}$          (d) $x = -1$          (e) no abs. max.

6. Where does the function $f(x) = e^{-x^2}$ have an absolute minimum?

(a) no abs. min.          (b) $x = -1/\sqrt{2}$          (c) $x = 1/\sqrt{2}$          (d) $x = 1$          (e) $x = 0$

7. Find the derivative of $f(x) = x^2 + (1/x^2)$.

(a) $2x - \dfrac{2}{x^3}$          (b) $2x + \dfrac{2}{x^3}$          (c) $2x - \dfrac{2}{x}$

(d) $2x + \dfrac{2}{x}$          (e) $x - \dfrac{1}{x}$

8. Let $f(x) = x^2 + 2x + 3$. Where does $f(x)$ have an inflection point?

(a) nowhere          (b) $x = 0$          (c) $x = 1$          (d) $x = 2$          (e) $x = 3$

9. Let $y = 2^x$ and compute $dy/dx$.

(a) $(\ln 2)2^x$          (b) $2^x$          (c) $x2^{x-1}$          (d) 0          (e) 2

10. The growth rate of a certain bacteria culture is proportional to its size, which means that it satisfies an exponential growth equation $y = Ae^{kt}$ where $t$ is the time and $y$ is the size of the population.. Suppose that the culture contains 10,000 bacteria at noon, and 40,000 bacteria at 2:00 PM. How many bacteria will it contain at 3:00 PM?

(a) 80,000          (b) 60,000          (c) $10,000 \cdot 2^{3/2}$          (d) $\dfrac{10,000}{\ln 4}$          (e) 120,000

11. If $f(x) = \dfrac{1}{x^2}$, which of the following limits equals $f'(x)$?

(a) $\displaystyle\lim_{h \to 0} \dfrac{1}{h}\left(\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}\right)$          (b) $\displaystyle\lim_{h \to 0} \dfrac{h}{(x+h)^2 - x^2}$

(c) $\displaystyle\lim_{h \to 0} h\left(\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}\right)$          (d) $\displaystyle\lim_{h \to 0} \dfrac{\frac{1}{x^2} - \frac{1}{h^2}}{h}$          (e) $\displaystyle\lim_{h \to 0} \dfrac{1}{h}\left(\dfrac{1}{x^2 - h^2}\right)$

12. Find the slope of the graph of $y = \sqrt{x}$ at the point $(9, 3)$.

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(a) 1/6                 (b) 1/3                 (c) 1/2                 (d) 3                 (e) 27

13. If $f(x) = \dfrac{x}{1+x^2}$, what is $f'(x)$?

(a) $\dfrac{1-x^2}{(1+x^2)^2}$          (b) $\dfrac{1}{2x}$          (c) $\dfrac{1}{(1+x^2)^2}$

(d) $\dfrac{1+3x^2}{1+x^2}$          (e) $\dfrac{1}{1+x^2}$

14. Which of the following is equal to $e^{(\ln 3)/2}$?

(a) $\sqrt{3}$          (b) $2^{1/3}$          (c) $2^{-1/3}$          (d) $\sqrt{e^3}$          (e) $3/2$

15. Suppose you put \$250 into a bank account that receives 8% annual interest compounded continuously. How many years will it take before your bank account has \$1,000?

(a) $\dfrac{\ln 4}{0.08}$          (b) $(0.08)\ln 4$          (c) $\dfrac{\ln(0.08)}{4}$          (d) $4\ln(0.08)$          (e) $250e^{0.08}$

16. Apply linear approximation to the function $f(x) = 1/x$ to estimate $1/2.016$. Write your answer in decimal form.

17. The graph of $y = f(x)$ is shown on the right.

Which of the following statements is true?

(a) $f(x)$ is decreasing and $f'(x)$ is increasing.
(b) $f(x)$ is increasing and $f'(x)$ is decreasing.
(c) $f(x)$ is increasing and $f'(x)$ is increasing.
(d) $f(x)$ is decreasing and $f'(x)$ is decreasing.
(e) None of the other statements is true.

18. Find the equation of the line passing through the points $(3, 7)$ and $(2, 2)$.

(a) $y = 5x - 8$          (b) $y = 7x - 14$          (c) $y = 7x - 12$

(e) $y = \dfrac{1}{5}x + \dfrac{8}{5}$          (e) $y = 5x + 2$

19. The derivative of $(z^4 + 3z^2 + 1)^2$ is

(a) $2(z^4 + 3z^2 + 1)(4z^3 + 6z)$          (b) $2(z^4 + 3z^2 + 1)$          (c) $(4z^3 + 6z)$

(d) $(4z^3 + 6z)^2$          (e) $2(4z + 6z)$

20. Suppose $f(x)$ is a function satisfying the following:

$$f'(1) = -2; \qquad f'(2) = -1; \qquad f'(3) = 1; \qquad f'(4) = 3$$

If $h(t) = f(t^3 + 1)$, what is $h'(1)$?

(a) $-3$          (b) $3$          (c) $1$          (d) $-6$          (e) $-1$

21. Find the equation of the line tangent to the curve $y = 1 + \ln x$ at the point $(e, 2)$.

(a) $y = \dfrac{x}{e} + 1$          (b) $y = ex + 2$          (c) $ey = x - 2$

(d) $y - 2 = x - e$          (e) $y = ex - 2$

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22. A manufacturer's monthly revenue from the sale of $x$ items is $R(x) = 20\sqrt{100x - x^2}$. Find the marginal revenue.

23. Consider the function $f(x) = (8 - 2x)\, e^{(x+7)}$. Which one of the following statements is true?

(a) $x = 3$ gives an absolute maximum.

(b) $x = 3$ gives a relative minimum.

(c) $x = 4$ gives a relative maximum.

(d) The function is everywhere concave down.

(e) There are no relative maxima or minima.

24. What is $\displaystyle\lim_{n \to \infty} \left(1 + \frac{2}{n}\right)^n$?

(a) $e^2$          (b) $\ln 2$          (c) $1$          (d) $0$          (e) $1/2$

25. For each of the following functions, determine where it is increasing, decreasing, concave up, concave down, has rel. max or min, has an inflection point. Then draw a rough sketch of the graph.

(i) $f(x) = x^2 e^{-x}$          (ii) $f(x) = 2x^3 - 3x^2 - 12x + 5$          (iii) $f(x) = x - \ln x$

26. Find $f'(x)$ in each of the following cases. Simplify your answer as much as possible.

(i) $f(x) = xe^{x^2}$                             (ii) $f(x) = \dfrac{\ln x}{x^2}, \quad x > 0.$

27. Where, if anywhere, is the following function not continuous?

$$f(x) = \begin{cases} x - 2 & \text{if } x \leq 0 \\ 2x - 2 & \text{if } 0 < x \leq 1 \\ 1 - x & \text{if } 1 < x < 2 \\ x & \text{if } x \geq 2 \end{cases}$$

(a) only at $x = 2$          (b) only at $x = 1$          (c) only at $x = 0$

(d) at $x = 1$ and $x = 2$          (e) at $x = 0$ and $x = 1$

29. Suppose $f(x)$ is a function defined for $x > 0$ whose first and second derivatives are as follows:

$$f'(x) = \frac{(x-1)(x+1)}{x^2} \qquad \text{and} \qquad f''(x) = \frac{2}{x^3}.$$

Which of the following might be the graph of $f$? (Use the letter to the lower left of the correct picture.)

minipage[b]2in (a)