[12pt]report
document Math 105

## Review questions - test 1

1. Find the slope of each of the following lines:
(a) through $(1,-1)$ and $(3,2)$
(b) given by the equation $2 x+3 y=5$
(c) through $(1,-2)$, parallel to the line with equation $x-2 y=1$.
(d) horizontal, through $(-4,7)$
2. Find the equation of each of the following lines:
(a) Through $(1 / 2,1)$ with slope $-2 / 3$.
(b) Through $(0,-1)$ and $(-2,5)$
(c) Through $(1,1)$ and $(-1,3)$
(d) Through $(-3,-1)$ and $(5,-1)$
(e) Through $(-3,-1)$ and $(-3,2)$.
(f) Through the point $(4,-1)$ and parallel to the line $2 x+5 y=6$.
3. Determine whether each of the following functions is continuous. If not, find the discontinuities.
(a) $f(x)= \begin{cases}2 x-1 & \text { if } x<0 \\ 1-x & \text { if } 0 \leq x \leq 1 \\ x-1 & \text { if } x>1\end{cases}$
(b) $f(x)= \begin{cases}1-x^{2} & \text { if } x<1 \\ x-1 & \text { if } x \geq 1\end{cases}$
(c) $f(x)= \begin{cases}x^{2}+1 & \text { if } x \leq 0 \\ 1 & \text { if } 0<x<1 \\ x^{2} & \text { if } x \geq 1\end{cases}$
(d) $f(x)= \begin{cases}x^{2} & \text { if } x \leq 0 \\ 1 & \text { if } 0<x<1 \\ x^{2}+1 & \text { if } x \geq 1\end{cases}$
4. Find the natural domain of each of the following functions.
(a) $f(x)=\sqrt{x-2}$
(b) $f(x)=\frac{1}{\sqrt{x-2}}$
(c) $f(x)=\frac{\sqrt{x}}{x-2}$
(d) $\frac{x}{x^{2}+4}$
(e) $\frac{x}{x^{2}-4}$
(f) $\frac{x}{\sqrt{x^{2}-4}}$
5. Find the vertical and horizontal asymptotes, if any, for the graphs of the following functions. If there are none, write "none."
(a) $f(x)=\frac{x^{2}-1}{x^{2}+1}$
(b) $f(x)=\frac{x}{x^{2}-4}$
(c) $f(x)=\frac{x^{4}}{x^{2}-100}$
(d) $f(x)=\frac{x+2}{3 x-4}$

$$
\text { (e) } f(x)=\frac{3 x^{4}+15 x^{3}+10 x^{2}+5 x+20}{x^{4}+1}
$$

6. A ball is thrown upward from a height of 15 feet with an initial speed of 8 feet per second. The height of the ball is given by the formula

$$
h=-16 t^{2}+8 t+15
$$

where $h$ is the height (in feet) and $t$ is the time (in seconds).
(a) When does the ball reach its highest point?
(b) How high does it go?
(c) When does it hit the ground?
7. Complete the squares of each of the following quadratics. Then state whether the graph has a maximum or a minimum point and find the coordinates of the point.
(a) $2 x^{2}+12 x+13$
(b) $-3 x^{2}+10 x-8$
8. A company that makes Adirondack chairs has fixed costs of $\$ 5,000$ per month plus a cost of $\$ 30$ per chair. The company plans to sell the chairs for $\$ 50$ each. Suppose they produce $q$ chairs a month, and assume that they sell every chair they produce.
(a) Find the monthly profit function.
(b) Find the break-even point.
9. A small company manufacturing tennis rackets has fixed costs of $\$ 700$ per week. For the first 50 rackets they make, each one costs $\$ 32$ to produce. Every racket over 50 they make costs $\$ 38$. If $x$ is the number of rackets they produce in a given week, find the cost function $C(x)$.
10. A company offers dinner cruises on the Chicago River. The company has found that the average number of passengers per night is 80 if the price is $\$ 45$ per person. At a price of $\$ 30$ per person, the average number of passengers per night is 125 . Let $p$ denote the price and $q$ denote the demand (the average number of passengers).
(a) Assuming that the demand is a linear function of the price, write the demand as a function of the price.
(b) Write the nightly revenue $R$ as a function of the price.
(c) What price should the company charge to maximize the revenue? (Round your answer off to the nearest dollar.)
11. Compute each of the following without a calculator. Your answer should be an integer or simple fraction.
(a) $\left(\frac{1}{9}\right)^{1 / 2}$
(b) $\left(\frac{1}{9}\right)^{-2}$
(c) $(0.1)^{3}$
(d) $(0.1)^{-3}$
12. Let $f(x)=\frac{1}{(x+2)^{2}}$. Compute the following.
(a) $f(1)$
(b) $f(0)$
(c) $f(a-1)$

## Answers :

1. (a) $3 / 2$ (b) $-2 / 3$ (c) $1 / 2$ (d) 0
2. (a) $y=-(2 / 3) x+(4 / 3)$
(b) $y=-3 x-1$
(c) $y=-x+2$
(d) $y=-1$
$\begin{array}{ll}\text { (e) } x=-3 & \text { (f) } 2 x+5 y=3\end{array}$
3. (a) disc. at $x=0$ (b) cont. (c) cont. (d) disc. at $x=0,1$
4. (a) $x \geq 2$ (b) $x>2$ (c) $x \geq 0, x \neq 2$ (d) all $x$ (e) $x \neq \pm 2$ (f) $|x|>2$
5. (a) h.a. $y=1$, no v.a. (b) h.a. $y=0$, v.a. $x=2, x=-2$
(c) no h.a., v.a. $x=10, x=-10 \quad$ (d) h.a. $y=1 / 3$, v.a. $x=4 / 3$
(e) h.a. $y=3$, no v.a.
6. (a) $1 / 4 \mathrm{sec} \quad$ (b) 16 ft (c) $5 / 4 \mathrm{sec}$
7. (a) $2(x+3)^{2}-5$, min at $(-3,-5)$
(b) $-3(x-(5 / 3))^{2}+(1 / 3), \max$ at $(5 / 3,1 / 3)$
8. (a) $P(x)=20 x-5000$ (b) 250
9. $C(x)= \begin{cases}700+32 x & \text { if } 0 \leq x \leq 50 \\ 2300+38(x-50) & \text { if } 50<x\end{cases}$
10. (a) $q=-3 p+215$ (b) $R=-3 p^{2}+215 p$ (c) 36
11. (a) $1 / 3$
(b) 81
(c) 0.001
(d) 1000
12. (a) $1 / 9$
(b) $1 / 4$
(c) $\frac{1}{(a+1)^{2}}$
