[12pt]report document **Math 105** 

## Review questions — test 1

1. Find the slope of each of the following lines:

- (a) through (1, -1) and (3, 2)
- (b) given by the equation 2x + 3y = 5
- (c) through (1, -2), parallel to the line with equation x 2y = 1.
- (d) horizontal, through (-4,7)

2. Find the equation of each of the following lines:

- (a) Through (1/2, 1) with slope -2/3.
- (b) Through (0, -1) and (-2, 5)
- (c) Through (1,1) and (-1,3)
- (d) Through (-3, -1) and (5, -1)
- (e) Through (-3, -1) and (-3, 2).
- (f) Through the point (4, -1) and parallel to the line 2x + 5y = 6.

3. Determine whether each of the following functions is continuous. If not, find the discontinuities.

(a) 
$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 0\\ 1 - x & \text{if } 0 \le x \le 1\\ x - 1 & \text{if } x > 1 \end{cases}$$

(b) 
$$f(x) = \begin{cases} 1 - x^2 & \text{if } x < 1\\ x - 1 & \text{if } x \ge 1 \end{cases}$$

(c) 
$$f(x) = \begin{cases} x^2 + 1 & \text{if } x \le 0\\ 1 & \text{if } 0 < x < 1\\ x^2 & \text{if } x \ge 1 \end{cases}$$

(d) 
$$f(x) = \begin{cases} x^2 & \text{if } x \le 0\\ 1 & \text{if } 0 < x < 1\\ x^2 + 1 & \text{if } x \ge 1 \end{cases}$$

4. Find the natural domain of each of the following functions.

(a) 
$$f(x) = \sqrt{x-2}$$

(b) 
$$f(x) = \frac{1}{\sqrt{x-2}}$$

(c) 
$$f(x) = \frac{\sqrt{x}}{x-2}$$

(d) 
$$\frac{x}{x^2 + 4}$$

(e) 
$$\frac{x}{x^2 - 4}$$

(f) 
$$\frac{x}{\sqrt{x^2 - 4}}$$

5. Find the vertical and horizontal asymptotes, if any, for the graphs of the following functions. If there are none, write "none."

(a) 
$$f(x) = \frac{x^2 - 1}{x^2 + 1}$$

(b) 
$$f(x) = \frac{x}{x^2 - 4}$$

(c) 
$$f(x) = \frac{x^4}{x^2 - 100}$$

(d) 
$$f(x) = \frac{x+2}{3x-4}$$

(e) 
$$f(x) = \frac{3x^4 + 15x^3 + 10x^2 + 5x + 20}{x^4 + 1}$$

6. A ball is thrown upward from a height of 15 feet with an initial speed of 8 feet per second. The height of the ball is given by the formula

$$h = -16t^2 + 8t + 15$$

where h is the height (in feet) and t is the time (in seconds).

- (a) When does the ball reach its highest point?
- (b) How high does it go?
- (c) When does it hit the ground?

7. Complete the squares of each of the following quadratics. Then state whether the graph has a maximum or a minimum point and find the coordinates of the point.

(a) 
$$2x^2 + 12x + 13$$
 (b)  $-3x^2 + 10x - 8$ 

- 8. A company that makes Adirondack chairs has fixed costs of \$5,000 per month plus a cost of \$30 per chair. The company plans to sell the chairs for \$50 each. Suppose they produce q chairs a month, and assume that they sell every chair they produce.
  - (a) Find the monthly profit function.
  - (b) Find the break-even point.
- 9. A small company manufacturing tennis rackets has fixed costs of \$700 per week. For the first 50 rackets they make, each one costs \$32 to produce. Every racket over 50 they make costs \$38. If x is the number of rackets they produce in a given week, find the cost function C(x).
- 10. A company offers dinner cruises on the Chicago River. The company has found that the average number of passengers per night is 80 if the price is \$45 per person. At a price of \$30 per person, the average number of passengers per night is 125. Let p denote the price and q denote the demand (the average number of passengers).
- (a) Assuming that the demand is a linear function of the price, write the demand as a function of the price.
  - (b) Write the nightly revenue R as a function of the price.
- (c) What price should the company charge to maximize the revenue? (Round your answer off to the nearest dollar.)
- 11. Compute each of the following without a calculator. Your answer should be an integer or simple fraction.

(a) 
$$\left(\frac{1}{9}\right)^{1/2}$$
 (b)  $\left(\frac{1}{9}\right)^{-2}$  (c)  $(0.1)^3$  (d)  $(0.1)^{-3}$ 

12. Let  $f(x) = \frac{1}{(x+2)^2}$ . Compute the following.

(a) 
$$f(1)$$
 (b)  $f(0)$  (c)  $f(a-1)$ 

## **Answers**:

**1.** (a) 
$$3/2$$
 (b)  $-2/3$  (c)  $1/2$  (d) 0

**2.** (a) 
$$y = -(2/3)x + (4/3)$$

(b) 
$$y = -3x - 1$$
 (c)  $y = -x + 2$  (d)  $y = -1$  (e)  $x = -3$  (f)  $2x + 5y = 3$ 

**3.** (a) disc. at 
$$x = 0$$
 (b) cont. (c) cont. (d) disc. at  $x = 0, 1$ 

**4.** (a) 
$$x \ge 2$$
 (b)  $x > 2$  (c)  $x \ge 0$ ,  $x \ne 2$  (d) all  $x$  (e)  $x \ne \pm 2$  (f)  $|x| > 2$ 

**5.** (a) h.a. 
$$y = 1$$
, no v.a. (b) h.a.  $y = 0$ , v.a.  $x = 2$ ,  $x = -2$ 

(c) no h.a., v.a. 
$$x = 10$$
,  $x = -10$  (d) h.a.  $y = 1/3$ , v.a.  $x = 4/3$ 

(e) h.a. 
$$y = 3$$
, no v.a.

7. (a) 
$$2(x+3)^2 - 5$$
, min at  $(-3, -5)$ 

(b) 
$$-3(x - (5/3))^2 + (1/3)$$
, max at  $(5/3, 1/3)$ 

**8.** (a) 
$$P(x) = 20x - 5000$$
 (b) 250

**9.** 
$$C(x) = \begin{cases} 700 + 32x & \text{if } 0 \le x \le 50\\ 2300 + 38(x - 50) & \text{if } 50 < x \end{cases}$$

**10.** (a) 
$$q = -3p + 215$$
 (b)  $R = -3p^2 + 215p$  (c) 36

**12.** (a) 
$$1/9$$
 (b)  $1/4$  (c)  $\frac{1}{(a+1)^2}$