[12pt]report
document **Math 105** **Review questions — test 2**

Note: Unless instructions say otherwise , answers involving e or $\ln$ can be left that way. You do not have to use a calculator to get a decimal answer.

1. Find each of the following. Your answer should be an integer or fraction.

(i) $e^{-2\ln 3}$ (ii) $\ln\sqrt{e}$ (iii) $\ln(\ln e)$ (iv) $\sqrt{e^{\ln 4}}$

2. Which of the following is equal to $e^{-2\ln t}$?

(a) $\dfrac{1}{2t}$ (b) $\left(\dfrac{1}{2}\right)^t$ (c) $\sqrt{t}$ (d) $\dfrac{1}{t^2}$ (e) 2^t

3. Which of the following is equal to $\ln(e^2)+\ln(e^3)$?

(a) e (b) 6 (c) 8 (d) 9 (e) 5

4. If a is a positive number, which of the following is equal to $\sqrt{a}$?

(a) $e^{a/2}$ (b) $e^{a\ln 2}$ (c) $e^{-a\ln 2}$ (d) $e^{(1/2)\ln a}$ (e) $e^{\ln(a/2)}$

5. Solve each of the following for x.

(i) $e^{2x}=3$ (ii) $\ln(1+\ln x)=0$ (iii) $\ln(x^2)=8$

(iv) $e^{\ln(2x+3)}=7$ (v) $e^{x^2-2x-3}=1$ (vi) $\ln(x^3)=6$

6. Suppose \$10,000 is deposited in account paying an annual interest rate of 4%, compounded continuously. How long will it take for the account to grow to \$25,000?

7. A certain amount of money is put into an account paying 5% annual interest, compounded continuously. How long will it take the initial investment to triple?

8. The radioactive substance einsteinium-253 decays exponentially — in other words, the amount y remaining after t days is given by a formula of the form $y=Ae^{-kt}$, where A is the initial amount and k is the decay constant. Suppose an initial amount of 30 mg of the substance decays to 10mg in 11.7 days.

(i) Find the decay constant of einsteinium-253.

(ii) Find its half life.

(iii) If you start with 100mg of einsteinium-253, how much will there be after 20 days?

9. The radioactive substance Cesium-137 decays exponentially and has a half life of 30 years. If t denotes time (in years) and y denotes the amount of Cesium remaining from an initial amount of 100 mg, which of the following equations correctly expresses y as a function of t?

(a) $y=100e^{-\left((\ln 2)/30\right)t}$ (b) $y=100e^{\left(30/(\ln 2)\right)t}$ (c) $y=100e^{-2(\ln 30)t}$

(d) $y=50e^{(\log_2 30)t}$ (e) $y=\dfrac{10}{3}e^{-t\ln 2}$

10. A certain radioactive substance satisfies the exponential decay law, $y=Ae^{kt}$, where t is the time (in days) and y is the amount (in grams) at time t. Suppose an inital amount of 100 grams decays to 25 grams at the end of 10 days. Which of the following equals the amount remaining at the end of 15 days?

(a) $25e^{-5\ln 4}$ (b) $100e^{-5\ln 2}$ (c) $100e^{-(2/3)\ln 2}$

(d) $25e^{1.5\ln 2}$ (e) $100e^{-1.5\ln 4}$

11. An account paying 4% annual interest compounded continuously starts out with an initial deposit, after which no further deposits or withdrawals are made. How many years will it take for the initial amount to double?

(a) $\dfrac{-2}{\ln(0.04)}$ (b) $\dfrac{(0.04)}{\ln 2}$ (c) $\dfrac{\ln 2}{0.04}$

(d) $\dfrac{2}{\ln(0.04)}$ (e) the answer depends on the initial amount

12. Suppose you invest a certain amount in an account that offers continuously compounded interest and your money triples in exactly 14 years. (After the initial investment you do not add or take away from the account until that time.) What is the annual interest rate?

13. Find $f'(x)$ for each of the following functions directly from the basic defining formula for the derivative without using any other rules or short-cut formulas.

(i) $f(x) = (2x+1)^2$ (ii) $f(x) = \dfrac{1}{x^2+1}$ (iii) $f(x) = \sqrt{x+1}$

14. Find $f'(x)$ for each of the following functions using any rule we have had so far.

(i) $f(x) = \dfrac{x^2}{3} + \dfrac{3}{x^2}$ (ii) $f(x) = \ln(4x^3)$ (iii) $f(x) = \ln(1/\sqrt{x})$

(iv) $f(x) = x^5 - 3x^4 + 2x^3 + x^2 - 5x + 7$ (v) $f(x) = e^{3x}$

(vi) $f(x) = \sqrt{x} - \dfrac{3}{\sqrt{x}}$ (vii) $f(x) = 5^x$ (viii) $f(x) = \ln(7/x)$

15. Find the second derivative $f''(x)$ for each of the following functions using any rule we have had so far.

(i) $f(x) = 2x^3 - 5x^2 + 7x - 1$ (ii) $f(x) = \ln(x^2)$ (iii) $f(x) = x^2 + e^x$

(iv) $f(x) = \sqrt{x}$ (v) $f(x) = \ln x - x^2$ (vi) $f(x) = e^{2x}$

16. For the graph of $y = x^2$, which of the following equals the slope of the secant line between the point on the graph where $x = 1$ and the point on the graph where $x = 1 + h$?

(a) $\dfrac{1-h}{h}$ (b) $1+h$ (c) $2+2h$ (d) $2+h$ (e) $\dfrac{1-h^2}{h}$

17. Suppose $f(x) = \sqrt{x+1}$. Which of the following limits represents the slope of the graph of $y = f(x)$ at the point $(3, 2)$?

(a) $\lim\limits_{h\to 0} \dfrac{\sqrt{1+h}-1}{h}$ (b) $\lim\limits_{h\to 0} \dfrac{\sqrt{3+h}-3}{h}$ (c) $\lim\limits_{h\to 0} \dfrac{\sqrt{4+h}-2}{h}$

(d) $\lim\limits_{h\to 0} \dfrac{\sqrt{h}-3}{h}$ (e) $\lim\limits_{h\to 0} \dfrac{\sqrt{h}-\sqrt{3}}{h}$

18. If $f(x) = \sqrt{x}$, which of the following expressions is equal to $f'(x)$?

(a) $\lim_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{h}$ (b) $\lim_{h\to 0} \dfrac{\sqrt{x}-\sqrt{h}}{x-h}$ (c) $\lim_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{h}}{h}$

(d) $\lim_{h\to 0} \dfrac{\sqrt{x+h}-\sqrt{x}}{\sqrt{x}}$ (e) $\lim_{h\to 0} \dfrac{\sqrt{x}+\sqrt{h}}{\sqrt{x}-\sqrt{h}}$

19. If $f(x) = \dfrac{1}{x^3}$, what is $f'(2)$?

20. Find the slope of the graph of $y = x^2 + \dfrac{4}{x^2}$ at the point $(2, 5)$.

21. Find the slope of the graph of $y = 1 + \ln x$ at the point $(e, 2)$

22. Find the equation of the line tangent to the graph of $y = x^{3/2} - 1$ at the point $(4, 7)$.

23. For what x does the graph of $y = x + \ln x$ have slope 4?

24. Find the equation of the line tangent to the graph of $y = x + e^x$ at the point $(0, 1)$.

25. For what x does the graph of $y = x^2 + 3x + 5$ have slope 9?

26. At what point is the tangent line to the graph of $y = x^2 + 3x + 1$ parallel to the line $x + y = 1$?

27. If a ball is dropped from a height of 100 feet, its height after t seconds is given by the function $f(t) = -16t^2 + 100$.

(i) Find the average velocity of the ball during the time period from 1 to 3/2 seconds.

(ii) What is the instantaneous velocity of the ball after 1 second? After 3/2 seconds?

(iii) What is the acceleration of the ball after 1 second? After 3/2 seconds? After 1.762235 seconds?

28. If a ball is thrown upward from a height of 5 feet with an initial speed of 30 feet per second, its height after t seconds is given by the function $f(t) = -16t^2 + 30t + 5$.

(i) Find the average velocity of the ball during the time period between t and $t + h$ seconds.

(ii) Find the instantaneous velocity of the ball at the end of t seconds.

29. An object is moving along a straight line so that its distance s from a fixed reference point at time t is given by the formula

$$s = 20 - \sqrt{t} + 3t^{3/2}.$$

(i) Find its (instantaneous) velocity when $t = 1$ and when $t = 4$.

(ii) Find its average velocity between times $t = 1$ and $t = 4$

(iii) Find its acceleration when $t = 1$.

30. Suppose the weight of a tumor in a laboratory animal is given by the formula $w = 0.02 + 0.35t + 0.002t^2$, where t is the time in days. How fast is the weight increasing when $t = 3$? When $t = 10$?

31. Suppose you put \$100 into an account that pays 4% simple interest annually.

(a) If you leave the money in the account without making any further deposits or withdrawals, how much will you have at the end of t years. (Your answer should be a function of t.)

(b) Use a calculator to find the amount you will have at the end of 4 years and at the end of 10 years.

32. Compute each of the following without a calculator. Your answer should be an integer or simple fraction.

(a) $\left(\frac{1}{9}\right)^{1/2}$ (b) $\left(\frac{1}{9}\right)^{-2}$ (c) $(0.1)^3$ (d) $(0.1)^{-3}$

33. Suppose you put \$5000 into an account that pays 6% annual interest, and make no further deposits or withdrawals.

(a) If the interest is compounded 12 times a year, how much will be in the account at the end of 1 year?

(b) If the interest is compounded 4 times a year, how much will be in the account at the end of 3 years?

34. Suppose an account pays 5% annual interest. How much should you put in the account now in order to have \$1000 at the end of two years, if

(a) the interest is compounded once a year?

(b) the interest is compounded 6 times a year?

Answers

1. (i) 1/9 (ii) 1/2 (iii) 0 (iv) 2 **2.** d **3.** e **4.** d

5. (i) $\dfrac{\ln 3}{2}$ (ii) 1 (iii) $\pm e^4$ (iv) 2 (v) -1 and 3 (v) e^2

6. $\dfrac{\ln 2.5}{0.04}$ **7.** $\dfrac{\ln 3}{0.05}$

8. (i) $\dfrac{\ln 3}{11.7}$ (ii) $\dfrac{11.7 \ln 2}{\ln 3}$ (iii) $100e^{-20 \ln 3/11.7} \approx 15.29$

9. a **10.** e **11.** c **12.** $\dfrac{\ln 3}{14} \approx 0.0785$ or 7.85%

13. (i) $f'(x) = 8x + 4$ (ii) $f'(x) = \dfrac{-2x}{(x^2+1)^2}$ (iii) $f'(x) = \dfrac{1}{2\sqrt{x+1}}$

14. (i) $f'(x) = \dfrac{2x}{3} - \dfrac{6}{x^3}$ (ii) $f'(x) = \dfrac{3}{x}$ (iii) $f'(x) = -\dfrac{1}{2x}$

(iv) $f'(x) = 5x^4 - 12x^3 + 6x^2 + 2x - 5$ (v) $f'(x) = 3e^{3x}$

(vi) $f'(x) = \dfrac{1}{2\sqrt{x}} + \dfrac{3}{2x^{3/2}}$ (vii) $f'(x) = 5^x \ln 5$ (viii) $f'(x) = -\dfrac{1}{x}$

15. (i) $f''(x) = 12x - 10$ (ii) $f''(x) = -\dfrac{2}{x^2}$ (iii) $f''(x) = 2 + e^x$

(iv) $f''(x) = -(1/4)x^{-3/2}$ (v) $f''(x) = -\left(\dfrac{1}{x^2} + 2\right)$ (vi) $f''(x) = 4e^{2x}$

16. d **17.** c **18.** a **19.** $-3/16$

20. 3 **21.** $1/e$ **22.** $y = 3x - 5$ **23.** 1/3

24. $y = 2x + 1$ **25.** 3 **26.** $(-2, -1)$

27. (i) -40 (ii) -32, -48 (iii) -32 at all times

28. (i) $-32t - 16h + 30$ (ii) $-32t + 30$

29. (i) 35/4 when $t = 4$ and 4 when $t = 1$ (ii) 20/3 (iii) 5/2

30. 0.362 when $t = 3$, 0.39 when $t = 10$.

31. (a) $100(1.04)^t$ (b) 116.99, 148.02

32. (a) 1/3 (b) 81 (c) 0.001 (d) 1000

33. (a) \$5308.39, (b) \$5978.09

34. (a) \$900.16, (b) \$898.06