[12pt]report
document **Math 105** **Review questions — test 3**

1. Suppose a beaker of hot water is put in a room whose temperature is kept fixed at 65 degrees Fahrenheit. According to Newton's law of cooling, the temperature of the water is given by the formula

$$H = 65 + Ae^{kt}$$

where H is the temperature, t is the time in minutes, and A and k are constants. At the end of 30 minutes the temperature of the water is 165 degrees, and it is decreasing at the rate of 1 degree per minute. What are the constants k and A?

2. A hard-boiled egg at 98 degrees Celsius is put into a sink of water that is kept at a steady temperature of 18 degrees Celsius. After 5 minutes, the egg's temperature is found to be 38 degrees Celsius. How much longer will it take the egg to reach 20 degrees Celsius?

3. Charcoal from a tree killed in the volcanic eruption that formed Crater Lake in Oregon contained 44.5% of the radioactive carbon found in a living sample of the same wood. How long ago did the volcanic eruption take place? (Use 5730 years as the half life of radioactive carbon.)

4. Find the equation of the line tangent to each of the following graphs at the given point.

(a) $y = (2 - x^2)^{30}$ at $(1, 1)$. (b) $y = \dfrac{x}{x+1}$ at $(2, 2/3)$.

(c) $y = \sqrt{x^2 + x + 2}$ at $(1, 2)$.

5. Find $f'(x)$ in each of the following cases.

(a) $f(x) = x^3 e^x$ (b) $f(x) = \dfrac{3}{(x^2 + x + 1)^2}$ (c) $f(x) = x\sqrt{x^2 - 4}$

(d) $f(x) = 3^{2x+5}$ (e) $f(x) = 2^{1/x}$ (f) $f(x) = \dfrac{e^{2x}}{1 + e^{3x}}$

(g) $f(x) = \dfrac{2x+1}{x+5}$ (h) $f(x) = \sqrt{e^{3x} + 1}$ (i) $f(x) = \dfrac{1}{x \ln x}$

(j) $f(x) = \ln(4x^3)$ (k) $f(x) = \ln(1/\sqrt{x})$

6. If $f(0) = 4$ and $f'(0) = 2$, find the equation of the line tangent to the graph of $y = \sqrt{x + f(x)}$ at $(0, 2)$.

7. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius. If a spherical balloon is expanding at a rate of 2 cu. ft. per minute, how fast is its radius increasing when the radius is 3 feet? (Assume the balloon always maintains an exact spherical shape.)

8. Given the following table of values:

tabular—c——c—c—c—c— x $f(x)$ $g(x)$ $f'(x)$ $g'(x)$

Find the derivatives of the following functions at the points indicated:

(a) $f(x) \cdot g(x)$ at $x = 3$ (b) $f(g(x))$ at $x = 1$ (c) $g(f(x))$ at $x = 1$

(d) $\dfrac{f(x)}{g(x)}$ at $x = 2$ (e) $\ln(f(x) + g(x))$ at $x = 1$ (f) $\dfrac{1}{x + f(x)}$ at $x = 2$

9. Suppose that the demand for a certain product is given by

$$q = 5000e^{-0.08p}$$

where p is the price and q is the number sold at that price.

(a) What quantity is sold when the price is \$10?

(b) At what rate is the demand changing with respect to the price when the price is \$10?

(c) What is the revenue when the price is \$10?

(d) At what rate is the revenue changing with respect to the price when the price is \$10?

10. Water is pouring into a hemispherical bowl of radius 10 inches. The volume of water satifies the equation $V = \frac{\pi}{3}(30x^2 - x^3)$, where x is the depth. Suppose that when the depth is 1 inch it is increasing at the rate of 0.5 inches per minute, and when the depth is 3 inches it is increasing at the rate of 0.2 inches per minute. How fast is the volume increasing at each of those times?

11. The demand for "I Love Kramer" tee shirts satisfies a function of the form $q = f(p)$, where p is the price and q is the number sold. The revenue is given by $R = pq$. Given that $f(12) = 6000$ and $f'(12) = -400$, find dR/dp when $p = 12$.

12. Determine where each of the functions is increasing and where it is decreasing, and find any relative maxima or minima.

(a) $f(x) = 3x^2 - 6x - 9$

(b) $f(x) = x + (4/x),\ x \neq 0$

(c) $f(x) = x^2 e^{-x}$

(d) $f(x) = x \ln x,\ x > 0$

(e) $f(x) = \dfrac{x^2}{x^2+4}$

(f) $f(x) = \dfrac{x}{x^2+4}$

13. Find the absolute maximum or minimum value, as indicated, of each of the following functions. Justify your answer by a few words of explanation.

(a) $f(x) = xe^{-2x}$, abs. max.

(b) $f(x) = 2x - \ln x,\ x > 0$, abs. min.

(c) $f(x) = 3x^2 - 4x$, abs. min.

14. Find the slope of the graph of $y = 1 + \ln x$ at the point $(e, 2)$.

15. Find where each of the following functions is increasing, where it is decreasing, and where its relative maxima and/or minima (if any) are.

(a) $f(x) = \ln(1 + x^2)$

(b) $f(x) = x^3 - 3x + 7$

16. Show that $e^x \geq 1 + x$ for all x by the following steps:

(a) Find where the function $f(x) = e^x - x$ has a relative minimum and explain why the function must have an absolute minimum there.

(c) Find the absolute minimum value of $f(x)$ and use that information to show that $e^x \geq 1 + x$ for all x.

17. For what x does the graph of $y = x + \ln x$ have slope 4?

18. Find the equation of the line tangent to the graph of $y = x + e^x$ at the point $(0, 1)$.

19. Let $C(x)$ be the cost of producing x units of a certain commodity. Which of the following procedures will give you the marginal cost of producing 100 units?

(a) compute $C(100)$

(b) compute $C'(100)$

(c) compute $C(100) - C(0)$

(d) set $C'(x) = 100$ and solve for x

(e) set $C(x) = 100$ and solve for x

20. A manufacturing company determines that the revenue obtained from the sale of x items is given by $R(x) = 220x - 4x^2$ and the cost of producing these items by $C(x) = 900 + 40x$. For what output level x does the marginal revenue equal the marginal cost? Is this the same as the break-even point?

21. Suppose the cost of producing x pounds of fertilizer is given by the cost function $\mathrm{C(x)} = 0.5\mathrm{x}^2 + 1.5x + 8$.

(i) Find the cost of producing 10 pounds.

(ii) Find the marginal cost for $x = 10$.

(iii) Approximate the extra cost in going from 10 pounds to 11 pounds.

22. Use linear approximation with the function $f(x) = \sqrt{x}$ to estimate:

(i) $\sqrt{50}$

(ii) $\sqrt{83}$

(iii) $\sqrt{98}$

23. Use linear approximation with an appropriate choice of function (and without a calculator) to estimate:

(i) $\dfrac{1}{0.98}$ (ii) $\dfrac{1}{2.108}$ (iii) $(8.1)^{1/3}$

24. Suppose the weight W of a growing puppy is a function of time t in days and you know that after 30 days, $W = 1500$ grams and $dW/dt = 4$ grams per day. Approximate the puppy's weight in grams at the end of 32 days.

25. Suppose the profit function for a certain manufactured item is given by

$$P(x) = 50x - 0.04x^2 - 10,000.$$

(i) Find the profit at production level $x = 600$.
(ii) Find the marginal profit at production level $x = 600$.
(iii) Use the previous answers to approximate the profit at production level $x = 601$ without computing $P(601)$.

Answers

1. $k = -0.01,\ A = 100e^{0.3}$
2. 13.3 minutes
3. 6693 years
4. (a) $y = -60x + 61$ (b) $y = \dfrac{1}{9}x + \dfrac{4}{9}$ $y = \dfrac{3}{4}x + \dfrac{5}{4}$
5. (a) $x^2e^x(x+3)$ (b) $\dfrac{-6(2x+1)}{(x^2+x+1)^3}$ (c) $\dfrac{2x^2-4}{\sqrt{x^2-4}}$

   (d) $(2\ln 3)3^{2x+5}$ (e) $-\dfrac{2^{1/x}\ln 2}{x^2}$ (f) $\dfrac{e^{2x}(2-e^{3x})}{(1+e^{3x})^2}$

   (g) $\dfrac{9}{(x+5)^2}$ (h) $\dfrac{3e^{3x}}{2\sqrt{e^{3x}+1}}$ (i) $-\dfrac{1+\ln x}{(x\ln x)^2}$

   (j) $f'(x) = \dfrac{3}{x}$ (k) $f'(x) = -\dfrac{1}{2x}$
6. $y = \dfrac{3}{4}x + 2$
7. $\dfrac{1}{18\pi}$
8. (a) -1 (b) $-1/2$ (c) $-3/10$

   (d) -30 (e) $-1/2$ (f) $-2/49$
9. (a) $5000e^{-0.8} \approx 2247$ (b) $-400e^{-0.8} \approx -180$

   (c) $50,000e^{-0.8} \approx 22,466$ (d) $1000e^{-0.8} \approx 449.33$
10. At $x = 1$, $dV/dt = 9.5\pi$. At $x = 3$, $dV/dt = 10.2\pi$
11. 1,200
12. (a) inc if $x > 1$, dec if $x < 1$, rel min at $x = 1$.

    (b) inc if $x < -2$ or $x > 2$, dec if $-2 < x < 0$ or $0 < x < 2$, rel min at $x = 2$, rel max at $x = -2$.

    (c) inc if $0 < x < 2$, dec if $x < 0$ or $x > 2$, rel min at $x = 0$, rel max at $x = 2$.

    (d) inc if $x > 1/e$, dec if $0 < x < 1/e$, rel min at $x = 1/e$.

    (e) inc if $x > 0$, dec if $x < 0$, rel min at $x = 0$.

    (f) inc if $-2 < x < 2$, dec if $x < -2$ or $x > 2$, rel min at $x = -2$, rel max at $x = 2$.
13. (a) $\dfrac{1}{2e} \approx 0.184$ (b) $1 + \ln 2 \approx 1.69$ (c) $-4/3$
14. $1/e$

15. (a) inc if $x > 0$, dec if $x < 0$, rel min at $x = 0$.
    (b) inc if $x < -1$ or $x > 1$, dec if $-1 < x < 1$,
    rel min at $x = 1$, rel max at $x = -1$.
16. Absolute min at $x = 0$, minimum value $= 1$.
17. 1/3
18. $y = 2x + 1$
19. b
20. 45/2, not the break-even point
21. (i) 73 (ii) 11.5 (iii) 11.5
22. (i) $\frac{99}{14} \approx 7.07143$ (ii) $\frac{82}{9} \approx 9.1111$ (iii) 9.9
23. (i) 1.02 (ii) $0.5 - 0.027 = 0.473$ (iii) $2 + \frac{0.1}{12} \approx 2.0083$
24. 1508
25. (i) $P(600) = 5,600$ (ii) $P'(600) = 2$ (iii) 5,602