Math 105 **Answers to supplementary problems**

1. $q = -30p + 1250$, with $p =$ price and $q =$ number of customers (demand).

2. (a) $p = -(1/20)\,q + 150$

(b) $R = -(1/20)\,q^2 + 150q$ and $P = -(1/20)\,q^2 + 115q - 10,000$.

Break-even points: 2209.5, 90.52.

(c) $R = 3000p - 20p^2$ and $C = 115,000 - 700p$, so

$P = -20p^2 + 3700p - 115,000$. Break-even points: 39.52, 145.47

3. (a) $q = -8p + 700$ (b) $R = -8p^2 + 700p$

4. (a) $-(x-3)^2 + 20$, opens downward, high point at $(3, 20)$

(b) $3(x+1)^2 - 4$, opens upward, low point at $(-1, -4)$

(c) $2\,(x + (5/4))^2 - (25/8)$, opens upward, low point at $(-5/4, -25/8)$

(d) $-2\,(x - (1/4))^2 + (9/8)$, opens downward, high point at $(1/4, 9/8)$

5. (a) 4 (b) 2 (c) 64

6. (a) 75 (b) 92.5

7. 43.75.

8. (a) h.a. $y = 0$, v.a. $x = -1$. (b) h.a. $y = 0$, v.a. $x = 1$

(c) h.a. $y = 2$, v.a. $x = -4$ (d) h.a. $y = 0$, v.a. $x = 2$ and $x = -2$

(e) h.a. is $y = 1/3$, no v.a. (f) no h.a., v.a. is $x = 2$.

9. 44.25 at 35 mph, 708 at 140 mph.

10. With $x =$ amt. of gold purchased and $f(x) =$ fee,

$$f(x) = \begin{cases} 0.06x & \text{if } 50 \le x \le 300 \\ 12 + 0.02x & \text{if } 300 < x \le 600 \\ 15 + 0.015x & \text{if } 600 < x \end{cases}$$

The function is continuous.

11. (a) continuous (b) discontinuity at $x = 0$

(c) continuous (d) discontinuity at $x = 1$

12.

(a)

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	1	1.225	1.5	1.837	2.25	2.756	3.375	4.134	5.0625

(b)

x	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	1	0.775	0.6	0.465	0.36	0.279	0.216	0.167	0.130

13. (a) $t = \dfrac{\ln 7}{\ln 5} \approx 1.209$ (b) $t = \dfrac{\ln 10}{\ln 2} \approx 3.322$

(c) $t = \dfrac{\ln 100}{3} \approx 1.535$ (d) $t = \ln(8/5) \approx 0.47$

14. (a) -3 (b) $x + 1$ (c) x^2 (d) 2

15. (a) $e^{7 \ln 3}$ (b) $e^{x \ln 5}$ (c) $e^{5 \ln x}$

16. (a) $\dfrac{\ln 12}{\ln 5} \approx 1.544$ (b) $\dfrac{\ln 5}{\ln 12} \approx 0.6477$ (c) $\dfrac{1}{\ln 2} \approx 1.4427$

17. (a) $\dfrac{40}{1 + 11e^{-0.8}} \approx 6.731$ billion

(b) when $t = \dfrac{\ln 11}{0.08} \approx 29.97$ (that is, around 2,020)

18. $\dfrac{\ln(5/3)}{0.08} \approx 6.385$ years

19. (a) $5000e^{0.32} \approx 6885.64$ (b) $\dfrac{\ln(8/5)}{8} \approx 0.059$, or 5.9 %

20. (a) 5 (b) $2a + 3$ (c) -3 (d) $1/3$

21. (a) $2x + 1$ (b) $\dfrac{-2}{(2x+3)^2}$ (c) $\dfrac{-2}{x^3}$

22. (a) $\dfrac{5}{3}\, x^{2/3}$ (b) $-3x^{-4}$, or $\dfrac{-3}{x^4}$ (c) $-\dfrac{1}{2}\, x^{-3/2}$, or $-\dfrac{1}{2x^{3/2}}$

23. (a) 36 ft. (b) 50 ft./sec. (c) 34 ft./sec. (d) 18 ft/sec. (e) -14 ft./sec.

(f) At $t = 1$ second the ball is moving upward. At $t = 2$ seconds the ball is moving downward.

(g) At $t = 25/16$ seconds. The ball is at its highest point.

24. (a) $6t^2 + (3/t)$ (b) $2 + 4 + (1/x)$ (c) $-1/x$

25. (a) $1/x$ (b) $-1/x$ (c) $1/(5x)$
(d) $1/(2x)$ (e) $3/t$ (f) $-3/t$

26. (a) $(1/2, \ln(1/2))$ (b) $(3, \ln 3)$

27. (a) $y = x - 1$ (b) lin. approx. gives 0.1, 0.02, and -0.01; calculator gives 0.9531, 0.1980, and -0.01005.

28. (a) $3e^t - 2t + 5$ (b) $2e^x + \dfrac{1}{2x^{3/2}}$ (c) $e^x - \dfrac{1}{x}$

29. a) $(\ln 3, 3)$ b) $(\ln(\frac{1}{2}), \frac{1}{2})$ c) $(-1, -0)$

30. a) $f'(x) = \dfrac{1}{2}e^{x/2}$ b) $f'(x) = 4e^{4x} = 4(e^x)^4$ c) $f'(t) = -e^{-t} = -\dfrac{1}{e^t}$
d) $\dfrac{dy}{dt} = \dfrac{1}{2}e^{t/2} = \dfrac{1}{2}\sqrt{e^t}$ e) $f'(x) = 3e^{3x} - 10e^{2x} + 3e^x$ f) $f'(x) = 2e^{2x} - 6e^x$

31. a) $f'(t) = (\ln 3)3^t + 3t^2$ b) $\dfrac{dy}{dx} = \dfrac{\ln 2}{3}2^x - \dfrac{2}{3x^2}$
c) $f'(x) = (\ln 5)5^x + 5(\ln 4)4^x$

32. a) $(-\dfrac{\ln(\ln 2)}{\ln 2}, 2^{-\frac{\ln(\ln 2)}{\ln 2}}) \approx (.539, 1.443)$ b) $(-\dfrac{1}{\ln 2}, 0)$

33. At $t = 1$, $1.05 \ln(1.05)P_0 \approx .0512P_0$
At $t = 2$, $(1.05)^2 \ln(1.05)P_0 \approx .0538P_0$

34. Population is approximately 10,586,000, and decreasing at approximately 21,000 people per year.

35. a) $k = 2\ln(\frac{1}{3}) = -2\ln 3 \approx -2.20$
b) $\dfrac{dH}{dt} = 30ke^{kt} = -66e^{-2.2t}$. Negative — Why?
c) When $t = 0$. Why?

36. $A = -80e^{\frac{3}{4}} \approx -169.36$ $B = -1/40 = -.025$

37. a) demand is 15,000 b) 1000 c) positive; increased

38. a) $R = 1000pe^{-0.02p}$ b) $\frac{dR}{dq} = -50(\ln(\frac{q}{1000}) + 1)$, or equivalently, $\frac{dR}{dq} = \frac{1 - 0.02p}{-0.02}$.

c) revenue is $10,000e^{-0.2} \approx \$8187$, marginal revenue is -40 dollars per unit

39. a) crit points: $x = -3, 2$; increasing on $(-\infty, -3)$ and $(2, \infty)$; decreasing on $(-3, 2)$; relative max at $x = -3$; relative min at $x = 2$

b) crit points: $x = 0, 1$; increasing on $(1, \infty)$; decreasing on $(-\infty, 1)$; relative min at $x = 1$.

c) crit points: $x = -2, 0, 2$; increasing on $(0, \infty)$; decreasing on $(-\infty, 0)$; relative min at $x = 0$.

d) crit points: $x = 1$; increasing on $(1, \infty)$; decreasing on $(0, 1)$; relative min at $x = 1$

e) crit points: $x = -1$; increasing on $(-1, \infty)$; decreasing on $(-\infty, -1)$; relative min at $x = -1$.

f) crit points: $x = -2/5, 0$; increasing on $(-\infty, -2/5)$, $(0, \infty)$; decreasing on $(-2/5, 0)$; relative max at $x = -2/5$; relative min at $x = 0$.

40. one solution (no critical points)

41. You should find that the min of $f(x)$ occurs at $x = 1$.

42. a) You should find that the min of $f(x)$ occurs at $x = 2$

b)

c) no, why?