Math 105 Exam I answers.

1. Suppose that a baker can sell a cake for \$7, so the revenue function is $R(q) = 7q$. The baker estimates the cost of making q cakes to be $C(q) = 2q + 600$. What is the baker's profit function?

   The profit function is $R(q) - C(q) = (7q) - (2q + 600) = \underline{5q - 600}$.

2. What is the y-intercept of the line $-3x + 4y = 2$?

   Solve for y to get $y = \frac{3}{4}x + \frac{1}{2}$. Thus the y-intercept is $\underline{\frac{1}{2}}$.

3. What is the slope of the line through the points $(1, 1)$ and $(3, -5)$?

   The slope is rise over run, or difference in y-values over difference in x-values, which is $\frac{1-(-5)}{1-3} = \frac{6}{-2} = -3$. So the answer is $\underline{-3}$.

4. What is the equation of the line which goes through the point $(0, 3)$ and which is parallel to the line $y = -4x + 5$?

   The slope of the line is -4, so use the point-slope form for the equation of the line: $y - 3 = -4(x - 0)$. Rewrite to get $\underline{y = -4x + 3}$.

5. Let $f(x) = \dfrac{x-7}{x+1}$. Find $f(a + 2)$.

   Plug in $a + 2$ for x to get $\dfrac{a+2-7}{a+2+1} = \underline{\dfrac{a-5}{a+3}}$.

6. Let $f(x) = 2x^2 - 8x - 5$. Complete the square in this quadratic function. (We are looking for an answer of the form "$f(x) = A(x + B)^2 + C$".)

   Following the procedure discussed in class gives the answer: $\underline{2(x - 2)^2 - 13}$.

7. Find the domain of the function $f(x) = \sqrt{x + 3}$.

   Because of the square root, the quantity $x + 3$ can't be negative: $x + 3 \geq 0$. Solving for x gives the answer: the domain is $\underline{\text{all } x \text{ with } x \geq -3}$.

8. Compute $(1/8)^{2/3}$. Express your answer as a fraction.

   Raising something to the power 2/3 is the same as taking the cube root and then squaring the result. So $(1/8)^{2/3} = (\sqrt[3]{1/8})^2 = (1/2)^2 = 1/4$. So the answer is $\underline{1/4}$.

9. A local theater owner realizes that when she charges \$18 per ticket, she averages 200 people per night. When she raises prices to \$21, the attendance drops to 170. Let p denote the ticket price, and let q denote the demand (the attendance).

   (a) Assuming that the demand is a linear function of the price, write demand in terms of price.

   The answer is the equation of the line going through the two points $(p_1, q_1) = (18, 200)$ and $(p_2, q_2) = (21, 170)$. The slope of this line is $\frac{200-170}{18-21} = \frac{30}{-3} = -10$, so the equation is $q - 200 = -10(p - 18)$, or $\underline{q = -10p + 380}$.

   (b) If the theater holds 260 people, how much should the owner charge in order to fill the theater to capacity?

   Since the theater holds 260 people, set $q = 260$ and solve for p: $260 = -10p + 380$. Solving for p gives a price of $\underline{\$12}$.

10. A businessman selling widgets determines that the price p of widgets and the demand q for widgets are related by the function $p = -6q + 1300$. Determine the revenue function $R(q)$ in terms of the demand q.

    The key thing here is that $R = pq$. Since we have a formula for p in terms of q, just plug it in: $R = (-6q + 1300)q = \underline{-6q^2 + 1300q}$.

11. A baseball cap company has a profit function of $-100p^2 + 1800p - 7200$, where p is the price charged per cap. How much should they charge to maximize profit?

    Complete the square to find out which price maximizes the profit function: the result of completing the square is $-100(p-9)^2 + 900$, so the profit is maximized when the price p is $\underline{\$9}$.

12. Use the quadratic formula to solve the equation $2x^2 + x - 6 = 0$.

    The quadratic formula says the solutions are

    $$x = \frac{-1 \pm \sqrt{1 - 4(2)(-6)}}{4} = \frac{-1 \pm \sqrt{49}}{4} = \frac{-1 \pm 7}{4}.$$

    So there are two solutions: $\underline{x = -2 \text{ and } x = 3/2}$.

13. Consider the function $f(x) = \frac{2(x+2)}{(x-3)(x-1)}$.

    (a) Find the vertical asymptotes of $f(x)$, or write "none" if there aren't any.

    The vertical asymptotes are the places where the denominator is zero: $\underline{x = 3 \text{ and } x = 1}$.

    (b) Find the horizontal asymptotes of $f(x)$, or write "none" if there aren't any.

    Since the degree of the numerator is less than the degree of the denominator, then $\underline{y = 0}$ is the horizontal asymptote.

14. The line $x = 2$ is a vertical asymptote of the function $f(x) = \frac{x+1}{x-2}$.

    (a) Does this curve climb or fall as it approaches the asymptote from the left?

    The function can only change sign (from plus to minus or vice versa) at the vertical asymptote $x = 2$ or when the function is zero, at $x = -1$. Just to the left of the asymptote, then, I can figure out the sign of the function by plugging in any number between -1 and 2, say $x = 0$. Since $f(0) = -1/2$, then the function is negative, so the curve <u>falls</u> as it approaches the asymptote.

    (b) Does this curve climb or fall as it approaches the asymptote from the right?

    To the right of the asymptote, the function does not change sign, so I can plug in any number bigger than 2, say $x = 3$. Since $f(3) = 4$, then the function is positive, so it <u>climbs</u> as it approaches the asymptote from the right.

15. Let $f(x) = \frac{3x-4}{2x-1}$. Find the horizontal asymptotes of $f(x)$.

    Since the degree of the numerator is the same as the degree of the denominator, then the horizontal asymptote occurs at the ratio of the leading terms: at $\underline{y = 3/2}$.

16. A certain sling-shot will fire a ball straight up in the air at a speed of 160 feet/second. The height of the ball after t seconds is given by the formula $h(t) = -16t^2 + 160t$. What is the maximum height that the ball reaches?

    Complete the square to maximize this formula. Completing the square yields $-16(t-5)^2 + 400$. Therefore the maximum height is <u>400 feet</u>.

17. Consider the function

    $$f(x) = \begin{cases} 3x + 1 & \text{if } x \leq 0 \\ x^2 + 1 & \text{if } 0 < x \leq 1 \\ 2 - x^2 & \text{if } x > 1. \end{cases}$$

    Is the function continuous? If not, give the x-values for its discontinuities.

    Plug $x = 0$ into $3x + 1$ and $x^2 + 1$: you get 1 both times. Since you get the same answer, the function is continuous at $x = 0$. Plug $x = 1$ into $x^2 + 1$ and $2 - x^2$: you get 2 and 1, so the function is <u>discontinuous at $x = 1$</u>.