Math 105 Exam II answers.

1. Solve the following for x.

   (a) $e^{3x-2} = 1$

   Apply ln to both sides: on the left, you get $\ln(e^{3x-2}) = 3x - 2$, and on the right you get $\ln(1) = 0$. So $3x - 2 = 0$, so $\boxed{x = \frac{2}{3}}$.

   (b) $\ln(5e^{-2x}) = 0$

   By the properties of logarithms, the left side can be rewritten as $\ln(5e^{-2x}) = \ln(5) + \ln(e^{-2x}) = \ln(5) + (-2x)$. Now set this equal to 0 and solve for x: $\boxed{x = \frac{\ln(5)}{2}} \approx 0.805$.

2. Evaluate the following; your answer should be an integer or fraction.

   (a) $\log_7(\frac{1}{49})$

   $\log_7(\frac{1}{49}) = \log_7((7)^{-2}) = \boxed{-2}$.

   (b) $e^{\ln(2^3)}$

   $e^{\ln(2^3)} = 2^3 = \boxed{8}$.

3. What is the half-life of a substance with a decay constant of 0.635?

   By the formula from class, the half-life is ln(2) divided by the decay constant: $\boxed{\ln(2)/0.635}$ ≈ 1.09.

4. A substance decays exponentially, so that the amount after t days is given by the formula $y = Ae^{-kt}$. If exactly $\frac{1}{3}$ of the initial amount remains after four days, what is the decay constant k of the substance?

   If you start with an amount A, then after 4 days, you will have $\frac{A}{3}$ remaining. So take the equation $\frac{A}{3} = Ae^{-k4}$ and solve for k: cancel A from each side to get $\frac{1}{3} = e^{-4k}$. Apply ln to both sides to get $\ln(\frac{1}{3}) = -4k$. Solve for k: $k = -\frac{\ln(1/3)}{4}$. Using properties of logarithms, $\ln(1/3) = -\ln(3)$, so you can rewrite this as $\boxed{k = \frac{\ln(3)}{4}} \approx 0.275$.

5. You owe a loan shark \$800; he charges interest at a rate of 35%. What is the formula for the amount of money owed after t years in these cases?

   (a) If the interest is compounded weekly (52 times a year)?

   $\boxed{800(1 + \frac{0.35}{52})^{52t}}$

   (b) If the interest is compounded continuously?

   $\boxed{800e^{0.35t}}$

6. You deposit \$2000 into an account paying 7%, compounded continuously. How long will it take for the account to grow to \$3000?

   Solve this equation for t: $3000 = 2000e^{0.07t}$. Divide by 2000 and apply ln: $\ln(3/2) = 0.07t$, so $\boxed{t = \frac{\ln(3/2)}{0.07}} = \frac{\ln(3)-\ln(2)}{0.07} \approx 5.79$.

7. What is the definition of the derivative of the function $f(x)$? (We are looking for the formula involving a limit and h and ...)

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}$$

8. Compute the derivative $f'(x)$ of the function $f(x) = 3x^2 - \frac{2}{x} + \sqrt[3]{x}$ using any rule we have had so far.

   Rewrite $\frac{2}{x}$ as $2x^{-1}$, and rewrite $\sqrt[3]{x}$ as $x^{\frac{1}{3}}$. Then the answer is $\boxed{f'(x) = 6x + \dfrac{2}{x^2} + \dfrac{1}{3}x^{-\frac{2}{3}}}$

9. Consider the function $f(x) = 7x^2$.

   (a) What is the slope of the secant line to this curve between the points $(1, 7)$ and $(1+h, 7(1+h)^2)$? Simplify as much as possible.
   $$\frac{7(1+h)^2-7}{h} = \frac{7(1+2h+h^2)-7}{h} = \frac{14h+7h^2}{h} = \boxed{14+7\text{h}}.$$

   (b) What is the slope of the tangent line to this curve at the point $(1, 7)$?
   It's the limit of the answer for part (a), as h goes to zero: $\boxed{14}$.

10. For what *two* values of x does the curve $y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 20x + 3$ have slope zero?

    Take the derivative, set it equal to zero, and solve for x: $x^2 + x - 20 = 0$. Solve this using the quadratic formula (or by factoring) to get $\boxed{x = 4}$ and $\boxed{x = -5}$.

11. Let $y = x^{1/2} - 3$. What is the equation of the line tangent to this curve at the point $(1, -2)$?

    Take the derivative and plug in $x = 1$ to find the slope: $y' = \frac{1}{2}x^{-1/2}$, so the slope is $\frac{1}{2}$. It goes through the point $(1, -2)$, so the equation is $y - (-2) = \frac{1}{2}(x - 1)$, or $\boxed{y = \frac{1}{2}x - \frac{5}{2}}$.

12. Let $f(x) = x^4 - 6x^2 + 7x + 1$. Compute $f''(x)$ using any rule we have had so far.

    $f'(x) = 4x^3 - 12x + 7$, so $\boxed{f''(x) = 12x^2 - 12}$.

13. You throw a ball into the air at a rate of 80 ft./sec., so that its position (height) after t seconds is given by the formula $s(t) = -16t^2 + 80t$. What is the velocity of the ball after 2 seconds?

    Take the derivative of $s(t)$ and plug in $t = 2$: $s'(t) = -32t + 80$, so the velocity after 2 seconds is $\boxed{s'(2) = 16}$.

14. A particle moves along the x-axis so that its position after t seconds is $s(t) = 3t^3 + 2t + 1$.

    (a) What is the average velocity of the particle between times $t = 0$ and $t = 1$?
    Average velocity is (distance traveled) divided by time elapsed: $\frac{s(1)-s(0)}{1-0}$. $s(1) = 6$ and $s(0) = 1$, so this is $\frac{6-1}{1-0} = \boxed{5}$.

    (b) What is the instantaneous velocity of the particle when $t = 1$?
    Instantaneous velocity is $s'(t)$. In this case, that is $s'(t) = 9t^2 + 2$. Plug in $t = 1$ to get the answer: $\boxed{s'(1) = 11}$.

    (c) What is the acceleration of the particle when $t = 1$?
    Acceleration is given by $s''(t) = 18t$. Plug in $t = 1$ to get the answer: $\boxed{s''(1) = 18}$.