Math–106 Exam #3 Wed. Nov. 22, 1996

1. In the definite integral $\int_0^{\left(\frac{\pi}{2}\right)^2} \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$ let $u = \sqrt{x}$

The resulting definite integral in u is

(a) $2\int_1^{\frac{\pi}{2}} \cos u\,du$ (b) $\frac{1}{2}\int_1^{\frac{\pi}{2}} \cos u\,du$ (c) $\int_0^{\frac{\pi}{2}} \cos u\,du$

(d) $2\int_0^{\left(\frac{\pi}{2}\right)^2} \frac{\cos u}{u}\,du$ (e) $2\int_1^{\left(\frac{\pi}{2}\right)^2} \cos u\,du$

2. Find the substitution $u = g(x)$ which changes the integral

$\int (\cos 5x)\sqrt{1-\sin 5x}\,dx$ to the form $-\frac{1}{5}\int \sqrt{u}\,du$.

(a) $u = \sqrt{1-\sin 5x}$ (b) $u = \sin 5x$ (c) $u = \cos 5x$

(d) $u = \frac{1}{5}\sin 5x$ (e) $u = 1 - \sin 5x$

3. Which of the following functions is a solution of the differential equation $y'' = -y$?

(a) $y = e^{-t}$ (b) $y = \sin t$ (c) $y = -t^2$

(d) $y = -1$ (e) $y = -3 + t^2$

4. Find the constant solutions (if any exist) of the differential equation $y' = t^2(y^2 - 4)$

(a) $y = 4$ (b) $y = 2$ only (c) $y = 2$, and -2 (d) $y = -2$ only (e) $y = 0$, and 2

5. Solve the equation $e^y y' = 2t$

(a) $y = t^2 + c$ (b) $y = e^t + c$ (c) $y = \ln(t^2 + c)$

(d) $\ln y = t^2 + c$ (e) $y = e^{t^2 + c}$

6. Solve the initial value problem

$$2y' = \frac{1}{y}\left(4t^3 + 1\right) \qquad y(0) = 3$$

(a) $y = (t + 1)^4 + 2$ (b) $y = t^4 + t + 3$ (c) $y = t^4 + 3$

(d) $y = \sqrt{t^4 + t + 9}$ (e) $y = \ln(t^4 + 1) + 3$

7. Suppose that a function $f(t)$ satisfies the initial value problem $y' = y \cos t - 5$; $y(\pi) = 3$. Then $f'(\pi) =$

(a) $-\frac{17}{4}$ (b) 3 (c) -2 (d) 2 (e) -8

8. A function $f(t)$ satisfies a differential equation $y' = g(y)$. The graph of $z = g(y)$ is as sketched below.

The graph of the solution $f(t)$ has an inflection point somewhere on one of the following intervals. Which one?

(a) $(-1,0)$ (b) $(-2,-1)$ (c) $(2,3)$

(d) $\left(0,\frac{1}{5}\right)$ (e) $\left(-\frac{1}{3}, 0\right)$

9. For the differential equation of problem 8 locate the stable equilibrium states

(a) $y = 0$ and $y = 1$ (b) $y = 0$ only (c) $y = 0$ and $y = 2$

(d) $y = 2$ only (e) $y = -1$ only

10. For the differential equation of problem 8 the solution function f(t) is increasing on one of the following intervals. Which one?

(a) (0,2) (b) (–2,–1) (c) $\left(\frac{2}{5}, 2\right)$

(d) (–1,0) (e) $\left(-\frac{1}{2}, \frac{2}{5}\right)$

11. Which of the following sketches most closely resembles the solutions of the differential equation $y' = y(2 - y)$.

(a) (b) (c)

(d) (e)

The yearly salaries of 20 people working at Acme Bolt Company are given in the table below. Use this table for problems 12, 13 and 14.

Salary	25000	30000	50000	100000
frequency	10	5	4	1

12. The relative frequency of those workers earning $30,000 is (correct to 2 decimal places).

(a) 0.02 (b) 6 (c) 0.25 (d) 5 (e) 0.17

13. The average (mean) salary of all 20 employees is

(a) 27,500 (b) 40,000 (c) 28,000
(d) 31,500 (e) 35,000

14. The <u>median</u> salary for all 20 employees is

(a) 27,500 (b) 35,000 (c) 51,300
(d) 31,500 (e) 28,000

In a certain experiment the outcomes were 0, 1, 2 and 3 with relative frequencies as shown in the following table

outcome	relative frequency
0	1/8
1	3/8
2	3/8
3	1/8

It is easily checked that the mean of the outcome values is 3/2.

15. The variance of the outcomes of this experiment is

(a) $\frac{1}{4}$ (b) 1 (c) $\frac{1}{2}$ (d) $\frac{3}{4}$ (e) $\frac{5}{4}$

16. The standard deviation of the outcomes of problem 15 is

(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{3}{2}$ (d) $\frac{9}{16}$ (e) $\frac{\sqrt{3}}{5}$