Name: ______________________ **Teacher/Section:** ____________________

Math 108 - Calculus II for Business

Final Exam - Spring Semester 2000

Friday, May 12, 1:45-3:45 p.m.

This Examination contains **30** multiple choice problems with no partial credit. Each is worth 5 points for a total of 150 points. Books and notes are not allowed. You may use your calculator.

Record your answers to these problems by placing an $\times$ through one letter for each problem below:

1. a b c d e	**11.** a b c d e	**21.** a b c d e
2. a b c d e	**12.** a b c d e	**22.** a b c d e
3. a b c d e	**13.** a b c d e	**23.** a b c d e
4. a b c d e	**14.** a b c d e	**24.** a b c d e
5. a b c d e	**15.** a b c d e	**25.** a b c d e
6. a b c d e	**16.** a b c d e	**26.** a b c d e
7. a b c d e	**17.** a b c d e	**27.** a b c d e
8. a b c d e	**18.** a b c d e	**28.** a b c d e
9. a b c d e	**19.** a b c d e	**29.** a b c d e
10. a b c d e	**20.** a b c d e	**30.** a b c d e

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

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GOOD LUCK!

NO PARTIAL CREDIT, EACH PROBLEM IS WORTH 5 POINTS
(For each problem, mark the correct answer on the cover.)

1. Calculate

$$\int_1^e 16x^3 \ln x \, dx.$$

(a) $4e^3 - 4$ (b) $5e^4 - 1$ (c) $12e^4 + 4$ (d) $20e^4 - 4$ (e) $3e^4 + 1$

2. Calculate

$$\int_0^{\sqrt{7}} 8t(t^2+1)^{1/3} \, dt.$$

(a) 7 (b) 360 (c) 45 (d) 24 (e) 56

3. The graph of g consists of a straight line and a semicircle, as shown. Evaluate $\int_0^6 g(x)\,dx$.

(a) $4 - 4\pi$
(b) $4 + 2\pi$
(c) $4 + 4\pi$
(d) $4 - 2\pi$
(e) $4 + 8\pi$

4. The table gives the values of a function f. Use them to estimate $\int_0^3 f(x)\,dx$ using Riemann sums with 3 equal subintervals and right endpoints.

x	0	1	2	3
$f(x)$	9.3	9.0	8.3	6.5

(a) 23.8 (b) 27.8 (c) 27.3 (d) 21.9 (e) 26.6

5. Find the area of the region enclosed by the curves $y = x^2$ and $y = x + 2$.

(a) 5 (b) $\frac{9}{2}$ (c) $\frac{11}{2}$ (d) 6 (e) $\frac{13}{2}$

6. Find the average value of $g(x) = \sqrt{x}$ on the interval $[1, 4]$.

(a) $\frac{14}{3}$ (b) $\frac{14}{9}$ (c) $\frac{7}{3}$ (d) 1 (e) $-\frac{1}{4}$

7. If the marginal cost of manufacturing x units of a product is

$$MC(x) = 6x^2 - 30x + 50$$

(measured in dollars per unit) and the fixed start-up cost is

$$C(0) = \$3,000,$$

find the cost of producing the first 10 units.

(a) \$6,000 (b) \$3,000 (c) \$2,000 (d) \$5,000 (e) \$4,000

8. Given the demand curve $p = D(q) = 50 - q/20$ and supply curve $p = S(q) = 20 + q/10$, find the consumer surplus.

(a) 3000 (b) 9000 (c) 2000 (d) 10,000 (e) 1000

9. Suppose that money is deposited steadily into a savings account at the rate of \$10,000 per year. Find the balance (in dollars) after 10 years if the account is earning 5.5% interest compounded continuously.

(a) $\frac{10000}{.055}(e^{.55} - 1)$ (b) $\frac{10000}{.055}(1 - e^{-.55})$ (c) $\frac{10000}{.055}(e^{.055} - 1)$

(d) $\frac{10000}{.55}(e^{.55} - 1)$ (e) $\frac{10000}{.55}(1 - e^{-.055})$

10. Compute

$$\int_0^2 xe^{-x}\, dx.$$

(a) $1 + \dfrac{3}{e^2}$ (b) $1 - \dfrac{3}{e^2}$ (c) $1 - \dfrac{2}{e^2}$

(d) $1 + \dfrac{2}{e^2}$ (e) $1 - \dfrac{1}{e^2}$

11. A person retires at age 60 with an Individual Retirement Account balance of $\$3,000,000$ and then makes continuous withdrawals at the rate of $\$120,000$ per year. Assume an interest rate of 7% compounded continuously. Find the differential equation and the initial condition which describe the balance $M(t)$ in the account t years after retirement.

(a) $\dfrac{dM}{dt} = 0.07M - 120,000, \quad M(0) = 3,000,000$

(b) $\dfrac{dM}{dt} = 0.07M + 120,000, \quad M(0) = 3,000,000$

(c) $\dfrac{dM}{dt} = 3,000,000 - 0.07M, \quad M(0) = 120,000$

(d) $\dfrac{dM}{dt} = 0.07M - 120,000, \quad M(0) = 120,000$

(e) $\dfrac{dM}{dt} = 120,000 - 0.07M, \quad M(0) = 3,000,000$

12. Use the Trapezoidal Rule and the following data to estimate the value of the integral $\int_1^{1.6} y\, dx$.

x	1.0	1.2	1.4	1.6
y	4.9	5.4	5.8	6.2

(a) 3.05 (b) 3.25 (c) 3.35 (d) 3.15 (e) 2.95

13. The function $y(t)$ is the solution of the initial value problem

$$\frac{dy}{dt} = te^y, \quad y(1) = 0.$$

Find $y(\sqrt{2})$.

(a) 1 (b) $\ln(1/2)$ (c) $-3/2$ (d) $\ln 2$ (e) 0

14. A function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = y^2 - 4y - 5.$$

Find the **unstable** equilibrium solutions.

(a) $y = 5$ only (b) $y = 0$ only (c) Both $y = -1$ and $y = 5$ (d) $y = -1$ only

(e) There are no unstable equilibrium solutions.

15. A random variable X has probability density function $f(x) = \frac{3}{4}x(2-x)$, $0 \le x \le 2$. Find the probability that $0 \le X \le 1$.

(a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$ (e) $\frac{3}{8}$

16. Find the equation of the tangent plane to the graph of the function $z = f(x, y) = xy^2$ at the point $(1, 3)$.

(a) $z = 6x + 9y - 24$ (b) $z = 9x + 6y - 18$ (c) $z = 9x + 3y - 9$

(d) $z = 3x + 6y - 12$ (e) $z = 6x + 3y - 6$

17 Suppose the probability distribution table of a random variable X is as follows:

x	0	1	2
$\Pr(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Which of the following is the expected value $E(X)$ of X?

(a) $\frac{2}{3}$ (b) $\frac{7}{6}$ (c) 1 (d) $\frac{5}{6}$ (e) $\frac{3}{4}$

18. Suppose that the sample space of an experiment is $S = \{s_1, s_2, s_3, s_4, s_5\}$, and that $P(\{s_1, s_2\}) = 0.5$, and $P(\{s_3, s_4\}) = 0.2$. Find $P(\{s_5\})$.

(a) 0.2 (b) 0.4 (c) 0.3 (d) 0.1 (e) 0

19. Consider the experiment of tossing two fair six-sided dice. Let X be a random variable that associates to each outcome the sum of the top faces. Find $P(X \geq 11)$.

(a) $\frac{1}{18}$ (b) $\frac{2}{11}$ (c) $\frac{1}{6}$ (d) $\frac{1}{36}$ (e) $\frac{1}{12}$

20. What is the probability that the four children in a family will consist of two boys and two girls? (Assume that each child is equally likely to be a boy or a girl).

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$ (e) $\frac{7}{16}$

21. Determine the value of the constant c such that the function $f(x) = ce^{-2x}, \ x \geq 0$, is a probability density.

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\frac{1}{4}$ (e) 4

22. Let X be a continuous random variable with density function $f(x) = \frac{1}{9}x^2$, $0 \le x \le 3$. What is E(X)?

(a) $\frac{9}{4}$ (b) $\frac{5}{4}$ (c) $\frac{3}{2}$ (d) $\frac{7}{4}$ (e) 2

23. Let X be a continuous random variable with density function $f(x) = \frac{1}{3}$, $1 \le x \le 4$. What is Var(X)?

(a) $\frac{5}{6}$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{2}{3}$ (e) $\frac{1}{3}$

24. For the function $f(x, y) = ye^{xy}$, find $\frac{\partial^2 f}{\partial y \partial x}$.

(a) xy^2e^{xy} (b) $(2y + y^2)e^{xy}$ (c) $(y + xy^2)e^{xy}$

(d) $(y + y^2)e^{xy}$ (e) $(2y + xy^2)e^{xy}$

25. Use the method of Lagrange multipliers to determine the minimum value of the function $f(x, y) = x^2 + y^2$ subject to the constraint $3x + 4y = 50$.

(a) $\frac{25}{2}$ (b) 25 (c) 100 (d) 75 (e) 50

26. The profit P in thousands of dollars per month of a company producing x hundred units of product X and y hundred units of product Y per month is given by

$$P(x, y) = 8x + 18y - x^2 - 4xy - 5y^2 + 30.$$

Find the pair (x, y) which maximizes the company's profit.

(a) $(1, 1)$ (b) $(2, 2)$ (c) $(2, 1)$ (d) $(3, 2)$ (e) $(1, 2)$

27. Which of the following statements must be true of any solution of the system of simultaneous linear equations $2x + 3y - 4z = 0$, $3y - 2z = 1$ and $2y + z = 3$?

(a) $x = \frac{1}{2}$ (b) $x = \frac{3}{2}$ (c) $y = 2$ (d) $y = 0$ (e) $z = \frac{1}{2}$

28. What is the distance between the two points $(2,-2,3)$ and $(5,0,6)$?

(a) $\sqrt{18}$ (b) $\sqrt{19}$ (c) $\sqrt{20}$ (d) $\sqrt{22}$ (e) $\sqrt{21}$

29. Let $f(x,y) = e^{-x^2-y^2}$. The level curve $f(x,y) = e^{-4}$ is:

(a) a straight line (b) a circle of radius 4 (c) a circle of radius $\sqrt{2}$

(d) a circle of radius 2 (e) a parabola

30. It is given that $(-2,4)$ is a critical point of a function $f(x,y)$ satisfying

$\frac{\partial f}{\partial x}(-2,4) = 0 = \frac{\partial f}{\partial y}(-2,4)$, $\frac{\partial^2 f}{\partial x^2}(-2,4) = -9$, $\frac{\partial^2 f}{\partial y^2}(-2,4) = 5$, $\frac{\partial^2 f}{\partial x \partial y}(-2,4) = -11$

Use the given information about $f(x,y)$ to determine the nature of the critical point $(-2,4)$.

(a) $(-2,4)$ is a saddle point

(b) $f(x,y)$ takes a relative minumum at $(-2,4)$

(c) there are no other critical points

(d) $f(x,y)$ takes a relative maximum at $(-2,4)$

(e) the second derivative test is inconclusive