

Name: _____

Teacher/Section: _____

Math 108 - Calculus II for Business

Final Exam - Spring Semester 2000

Friday, May 12, 1:45-3:45 p.m.

This Examination contains **30** multiple choice problems with no partial credit. Each is worth 5 points for a total of 150 points. Books and notes are not allowed. You may use your calculator.

Record your answers to these problems by placing an \times through one letter for each problem below:

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| 1. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 11. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 21. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
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| 3. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 13. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 23. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
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| 5. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 15. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 25. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 6. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 16. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 26. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 7. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 17. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 27. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 8. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 18. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 28. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 9. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 19. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 29. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |
| 10. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 20. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e | 30. | <input type="checkbox"/> a | <input type="checkbox"/> b | <input type="checkbox"/> c | <input type="checkbox"/> d | <input type="checkbox"/> e |

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

GOOD LUCK!!!

NO PARTIAL CREDIT, EACH PROBLEM IS WORTH 5 POINTS
(For each problem, mark the correct answer on the cover.)

1. Calculate

$$\int_1^e 16x^3 \ln x \, dx.$$

- (a) $4e^3 - 4$ (b) $5e^4 - 1$ (c) $12e^4 + 4$ (d) $20e^4 - 4$ (e) $3e^4 + 1$

2. Calculate

$$\int_0^{\sqrt{7}} 8t(t^2 + 1)^{1/3} \, dt.$$

- (a) 7 (b) 360 (c) 45 (d) 24 (e) 56

3. The graph of g consists of a straight line and a semi-circle, as shown. Evaluate $\int_0^6 g(x) \, dx$.

- (a) $4 - 4\pi$
(b) $4 + 2\pi$
(c) $4 + 4\pi$
(d) $4 - 2\pi$
(e) $4 + 8\pi$

4. The table gives the values of a function f . Use them to estimate $\int_0^3 f(x) dx$ using Riemann sums with 3 equal subintervals and right endpoints.

x	0	1	2	3
$f(x)$	9.3	9.0	8.3	6.5

- (a) 23.8 (b) 27.8 (c) 27.3 (d) 21.9 (e) 26.6

5. Find the area of the region enclosed by the curves $y = x^2$ and $y = x + 2$.

- (a) 5 (b) $\frac{9}{2}$ (c) $\frac{11}{2}$ (d) 6 (e) $\frac{13}{2}$

6. Find the average value of $g(x) = \sqrt{x}$ on the interval $[1, 4]$.

- (a) $\frac{14}{3}$ (b) $\frac{14}{9}$ (c) $\frac{7}{3}$ (d) 1 (e) $-\frac{1}{4}$

7. If the marginal cost of manufacturing x units of a product is

$$MC(x) = 6x^2 - 30x + 50$$

(measured in dollars per unit) and the fixed start-up cost is

$$C(0) = \$3,000,$$

find the cost of producing the first 10 units.

- (a) \$6,000 (b) \$3,000 (c) \$2,000 (d) \$5,000 (e) \$4,000

8. Given the demand curve $p = D(q) = 50 - q/20$ and supply curve $p = S(q) = 20 + q/10$, find the consumer surplus.

- (a) 3000 (b) 9000 (c) 2000 (d) 10,000 (e) 1000

9. Suppose that money is deposited steadily into a savings account at the rate of \$10,000 per year. Find the balance (in dollars) after 10 years if the account is earning 5.5% interest compounded continuously.

- (a) $\frac{10000}{.055}(e^{.55} - 1)$ (b) $\frac{10000}{.055}(1 - e^{-.55})$ (c) $\frac{10000}{.055}(e^{.055} - 1)$
- (d) $\frac{10000}{.55}(e^{.55} - 1)$ (e) $\frac{10000}{.55}(1 - e^{-.055})$

10. Compute

$$\int_0^2 xe^{-x} dx.$$

(a) $1 + \frac{3}{e^2}$

(b) $1 - \frac{3}{e^2}$

(c) $1 - \frac{2}{e^2}$

(d) $1 + \frac{2}{e^2}$

(e) $1 - \frac{1}{e^2}$

11. A person retires at age 60 with an Individual Retirement Account balance of \$3,000,000 and then makes continuous withdrawals at the rate of \$120,000 per year. Assume an interest rate of 7% compounded continuously. Find the differential equation and the initial condition which describe the balance $M(t)$ in the account t years after retirement.

(a) $\frac{dM}{dt} = 0.07M - 120,000, \quad M(0) = 3,000,000$

(b) $\frac{dM}{dt} = 0.07M + 120,000, \quad M(0) = 3,000,000$

(c) $\frac{dM}{dt} = 3,000,000 - 0.07M, \quad M(0) = 120,000$

(d) $\frac{dM}{dt} = 0.07M - 120,000, \quad M(0) = 120,000$

(e) $\frac{dM}{dt} = 120,000 - 0.07M, \quad M(0) = 3,000,000$

12. Use the Trapezoidal Rule and the following data to estimate the value of the integral $\int_1^{1.6} y dx$.

x	1.0	1.2	1.4	1.6
y	4.9	5.4	5.8	6.2

(a) 3.05

(b) 3.25

(c) 3.35

(d) 3.15

(e) 2.95

13. The function $y(t)$ is the solution of the initial value problem

$$\frac{dy}{dt} = te^y, \quad y(1) = 0.$$

Find $y(\sqrt{2})$.

- (a) 1 (b) $\ln(1/2)$ (c) $-3/2$ (d) $\ln 2$ (e) 0

14. A function $y(t)$ satisfies the differential equation

$$\frac{dy}{dt} = y^2 - 4y - 5.$$

Find the **unstable** equilibrium solutions.

- (a) $y = 5$ only (b) $y = 0$ only (c) Both $y = -1$ and $y = 5$ (d) $y = -1$ only
(e) There are no unstable equilibrium solutions.

15. A random variable X has probability density function $f(x) = \frac{3}{4}x(2-x)$, $0 \leq x \leq 2$. Find the probability that $0 \leq X \leq 1$.

- (a) $\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) $\frac{1}{2}$ (e) $\frac{3}{8}$

16. Find the equation of the tangent plane to the graph of the function $z = f(x, y) = xy^2$ at the point $(1, 3)$.

- (a) $z = 6x + 3y - 6$ (b) $z = 6x + 9y - 24$ (c) $z = 9x + 3y - 9$
(d) $z = 3x + 6y - 12$ (e) $z = 9x + 6y - 18$

17 Suppose the probability distribution table of a random variable X is as follows:

x	0	1	2
$\Pr(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

Which of the following is the expected value $E(X)$ of X ?

- (a) $\frac{5}{6}$ (b) $\frac{7}{6}$ (c) 1 (d) $\frac{2}{3}$ (e) $\frac{3}{4}$

18. Suppose that the sample space of an experiment is $S = \{s_1, s_2, s_3, s_4, s_5\}$, and that $P(\{s_1, s_2\}) = 0.5$, and $P(\{s_3, s_4\}) = 0.2$. Find $P(\{s_5\})$.

- (a) 0.3 (b) 0.4 (c) 0.2 (d) 0.1 (e) 0

19. Consider the experiment of tossing two fair six-sided dice. Let X be a random variable that associates to each outcome the sum of the top faces. Find $P(X \geq 11)$.

- (a) $\frac{1}{6}$ (b) $\frac{2}{11}$ (c) $\frac{1}{12}$ (d) $\frac{1}{36}$ (e) $\frac{1}{18}$

20. What is the probability that the four children in a family will consist of two boys and two girls? (Assume that each child is equally likely to be a boy or a girl).

- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{5}$ (d) $\frac{3}{8}$ (e) $\frac{7}{16}$

21. Determine the value of the constant c such that the function $f(x) = ce^{-2x}$, $x \geq 0$, is a probability density.

- (a) 1 (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$ (e) 4

22. Let X be a continuous random variable with density function $f(x) = \frac{1}{9}x^2$, $0 \leq x \leq 3$. What is $E(X)$?

- (a) $\frac{9}{4}$ (b) $\frac{5}{4}$ (c) $\frac{3}{2}$ (d) $\frac{7}{4}$ (e) 2

23. Let X be a continuous random variable with density function $f(x) = \frac{1}{3}$, $1 \leq x \leq 4$. What is $\text{Var}(X)$?

- (a) $\frac{3}{4}$ (b) $\frac{5}{6}$ (c) 1 (d) $\frac{2}{3}$ (e) $\frac{1}{3}$

24. For the function $f(x, y) = ye^{xy}$, find $\frac{\partial^2 f}{\partial y \partial x}$.

- (a) $(2y + y^2)e^{xy}$ (b) $(2y + xy^2)e^{xy}$ (c) $(y + xy^2)e^{xy}$
(d) $(y + y^2)e^{xy}$ (e) xy^2e^{xy}

25. Use the method of Lagrange multipliers to determine the minimum value of the function $f(x, y) = x^2 + y^2$ subject to the constraint $3x + 4y = 50$.

- (a) $\frac{25}{2}$ (b) 100 (c) 25 (d) 75 (e) 50

26. The profit P in thousands of dollars per month of a company producing x hundred units of product X and y hundred units of product Y per month is given by

$$P(x, y) = 8x + 18y - x^2 - 4xy - 5y^2 + 30.$$

Find the pair (x, y) which maximizes the company's profit.

- (a) (1, 1) (b) (2, 2) (c) (2, 1) (d) (3, 2) (e) (1, 2)

27. Which of the following statements must be true of any solution of the system of simultaneous linear equations $2x + 3y - 4z = 0$, $3y - 2z = 1$ and $2y + z = 3$?

- (a) $z = \frac{1}{2}$ (b) $x = \frac{3}{2}$ (c) $y = 2$ (d) $y = 0$ (e) $x = \frac{1}{2}$

28. What is the distance between the two points $(2, -2, 3)$ and $(5, 0, 6)$?

- (a) $\sqrt{20}$ (b) $\sqrt{19}$ (c) $\sqrt{22}$ (d) $\sqrt{18}$ (e) $\sqrt{21}$

29. Let $f(x, y) = e^{-x^2-y^2}$. The level curve $f(x, y) = e^{-4}$ is:

- (a) a circle of radius 4 (b) a circle of radius 2
(c) a circle of radius $\sqrt{2}$ (d) a straight line (e) a parabola

30. It is given that $(-2, 4)$ is a critical point of a function $f(x, y)$ satisfying

$$\frac{\partial f}{\partial x}(-2, 4) = 0 = \frac{\partial f}{\partial y}(-2, 4), \quad \frac{\partial^2 f}{\partial x^2}(-2, 4) = -9, \quad \frac{\partial^2 f}{\partial y^2}(-2, 4) = 5, \quad \frac{\partial^2 f}{\partial x \partial y}(-2, 4) = -11$$

Use the given information about $f(x, y)$ to determine the nature of the critical point $(-2, 4)$.

- (a) the second derivative test is inconclusive
(b) $f(x, y)$ takes a relative minimum at $(-2, 4)$
(c) there are no other critical points
(d) $f(x, y)$ takes a relative maximum at $(-2, 4)$
(e) $(-2, 4)$ is a saddle point