

1. Compute the improper integral

$$\int_2^{\infty} x^{-4} dx.$$

(a)  $\frac{1}{24}$

(b)  $-\frac{1}{6}$

(c)  $\frac{1}{8}$

(d)  $\frac{1}{3}$

(e)  $-\frac{1}{12}$

2. Find the distance of the point  $P = (2, 4, -4)$  to the origin.

(a) 10

(b) 6

(c) 2

(d) 4

(e) 36

3. Solve the following differential equation with given initial condition:

$$y' = \frac{1}{2}y - 3, \quad y(0) = 4.$$

(a)  $f(t) = 6 - e^{\frac{1}{2}t}$       (b)  $f(t) = 4 - 2e^{\frac{1}{2}t}$       (c)  $f(t) = 6 - \frac{1}{2}e^{\frac{1}{2}t}$       (d)  $f(t) = 5 - e^{3t}$

(e)  $f(t) = 6 - 2e^{\frac{1}{2}t}$

4. Assume  $y' = t^2$  and  $y(0) = 1$ . Compute  $y(3)$ .

(a) 9      (b) 10      (c) 12      (d) 3      (e) 0

5. Find the equation of the plane through the points  $(2, 0, 0)$ ,  $(0, 4, 0)$  and  $(0, 0, 8)$ .

(a)  $z = 4 - 8x - 2y$       (b)  $z = 8 - 2x - 4y$       (c)  $z = \frac{1}{2} + \frac{1}{8}x - \frac{1}{4}y$       (d)  $z = \frac{1}{2} + \frac{1}{8}x + \frac{1}{4}y$

(e)  $z = 8 - 4x - 2y$

6. The rate of growth for a particular population is given by  $\frac{\partial p}{\partial t} = .6p - .003p^2$ . What is the population's carrying capacity?

(a) 300

(b) 200

(c) 100

(d) 600

(e) 50

7. Solve the differential equation  $y' = e^{t-y}$

(a)  $y = e^t + C$

(b)  $y = e^{t-y} + C$

(c)  $y = e^{t-y} + e^t + C$

(d)  $y = \ln(e^t + C)$

(e)  $y = \ln(e^t + t + C)$

8. Let  $f(x, y)$  be a function such that

$$f(1, -3) = 2, \quad \frac{\partial f}{\partial x}(1, -3) = -1, \quad \text{and} \quad \frac{\partial f}{\partial y}(1, -3) = 2.$$

Use linear approximation to estimate the value of  $f(1.1, -3.1)$ .

(a) 1.9

(b) 1.7

(c) 2.1

(d) 2.3

(e) 1.5

9. Find the only critical point of the function

$$f(x, y) = 3x^2 + y^2 + 2xy - 8x + 4y + 1.$$

- (a)  $(0, 0)$       (b)  $(3, -5)$       (c)  $(4, -8)$       (d)  $(0, 4)$       (e)  $(4, -6)$

10. Find the maximum of the function  $f(x, y) = x + 2y$  subject to the constraint  $x^2 + y^2 = 1$ .

- (a)  $(\sqrt{5}, \sqrt{10})$       (b)  $(-\sqrt{5}/5, -2\sqrt{5}/5)$       (c)  $(\sqrt{5}/5, 2\sqrt{5}/5)$       (d)  $(-\sqrt{5}/5, 2\sqrt{5}/5)$   
(e)  $(\sqrt{5}/5, -2\sqrt{5}/5)$

11. Find the line of least squares  $y = ax + b$  which best fits the data points  $(-3, -5)$ ,  $(1, 3)$  and  $(2, 5)$ .

- (a)  $y = 5x + 4$       (b)  $y = x + 1$       (c)  $y = 3x$       (d)  $y = 3x + 2$       (e)  $y = 2x + 1$

12. Compute  $\frac{\partial f}{\partial x}(0, 1)$  for the function  $f(x, y) = x^2e^{3x+y} + 5ye^y + 2xy$ .

- (a) 5                      (b) 3                      (c) 4                      (d) 0                      (e) 2

13. Consider the function  $f(x, y) = -6x^2 - 7xy - 2y^2$ . Which of the following statements is true?

- (a)  $(0, 0)$  is a saddle point.                      (b)  $(0, 0)$  is a local maximum.  
(c) The second derivative test is inconclusive.                      (d)  $(0, 0)$  is a local minimum.  
(e) There are no critical points.

14. (11 pts) A person opens an Individual Retirement Account (IRA) with the initial amount of \$50,000. Then \$6,000 per year is deposited in this IRA in a uniform and continuous manner. Assume that the interest rate is 7.5% compounded continuously.

(4 pts) Model this problem as a Calculus problem by finding a **differential equation** and an **initial condition** describing the amount of money,  $M(t)$ , in the IRA at any time  $t$ .

(4 pts) Solve the obtained differential equation, i.e. find  $M(t)$  at any time  $t$ .

(3 pts) Compute the balance in the IRA after 20 years. (Simplify your answer as much as possible.)

15. (11 pts) The Cobb-Douglas production function for a certain product is given by  $f(x, y) = 5x^{\frac{3}{5}}y^{\frac{2}{5}}$  where  $f(x, y)$  is the quantity produced,  $x$  denotes units of labor force and  $y$  denotes units of capital. Assume that each unit of labor costs \$400, each unit of capital costs \$100, and the total budget is \$20,000. Find the amounts of labor and capital which will maximize the company's production while keeping within the constraints of the budget. (**Remark:** No second derivative test required).

Answer:

$x =$

$y =$



Math 108, Test 2

October 31, 2002

1. Please cross  the correct answers.
2. This test will be exactly 75 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 8 PAGES with 15 PROBLEMS. The point value for each multiple choice problem is 6 points. Problems 14 and 15 are worth 11 points each. Use the back of the test pages for scratch work.
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Name: \_\_\_\_\_

1.  a  b  c  d  e

2.  a  b  c  d  e

3.  a  b  c  d  e

4.  a  b  c  d  e

5.  a  b  c  d  e

6.  a  b  c  d  e

7.  a  b  c  d  e

8.  a  b  c  d  e

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11.  a  b  c  d  e

12.  a  b  c  d  e

13.  a  b  c  d  e

Mult. Choice \_\_\_\_\_

14 \_\_\_\_\_

15 \_\_\_\_\_

Total \_\_\_\_\_

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2.  a  b  c  d  e

3.  a  b  c  d  e

4.  a  b  c  d  e

5.  a  b  c  d  e

6.  a  b  c  d  e

7.  a  b  c  d  e

8.  a  b  c  d  e

9.  a  b  c  d  e

10.  a  b  c  d  e

11.  a  b  c  d  e

12.  a  b  c  d  e

13.  a  b  c  d  e

Mult. Choice \_\_\_\_\_

14 \_\_\_\_\_

15 \_\_\_\_\_

Total \_\_\_\_\_

Sign your name:

1. Compute the improper integral

$$\int_2^{\infty} x^{-4} dx.$$

(a)  $\frac{1}{3}$

(b)  $\frac{1}{8}$

(c)  $-\frac{1}{12}$

(d)  $\frac{1}{24}$

(e)  $-\frac{1}{6}$

2. Find the distance of the point  $P = (2, 4, -4)$  to the origin.

(a) 2

(b) 4

(c) 36

(d) 10

(e) 6

3. Solve the following differential equation with given initial condition:

$$y' = \frac{1}{2}y - 3, \quad y(0) = 4.$$

(a)  $f(t) = 6 - e^{\frac{1}{2}t}$       (b)  $f(t) = 4 - 2e^{\frac{1}{2}t}$       (c)  $f(t) = 5 - e^{3t}$       (d)  $f(t) = 6 - \frac{1}{2}e^{\frac{1}{2}t}$

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4. Assume  $y' = t^2$  and  $y(0) = 1$ . Compute  $y(3)$ .

(a) 3      (b) 0      (c) 10      (d) 12      (e) 9

5. Find the equation of the plane through the points  $(2, 0, 0)$ ,  $(0, 4, 0)$  and  $(0, 0, 8)$ .

(a)  $z = 4 - 8x - 2y$       (b)  $z = \frac{1}{2} + \frac{1}{8}x + \frac{1}{4}y$       (c)  $z = 8 - 4x - 2y$       (d)  $z = \frac{1}{2} + \frac{1}{8}x - \frac{1}{4}y$

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6. The rate of growth for a particular population is given by  $\frac{\partial p}{\partial t} = .6p - .003p^2$ . What is the population's carrying capacity?

(a) 200

(b) 50

(c) 600

(d) 100

(e) 300



9. Find the only critical point of the function

$$f(x, y) = 3x^2 + y^2 + 2xy - 8x + 4y + 1.$$

- (a)  $(4, -6)$       (b)  $(0, 0)$       (c)  $(0, 4)$       (d)  $(4, -8)$       (e)  $(3, -5)$

10. Find the maximum of the function  $f(x, y) = x + 2y$  subject to the constraint  $x^2 + y^2 = 1$ .

- (a)  $(-\sqrt{5}/5, 2\sqrt{5}/5)$       (b)  $(\sqrt{5}/5, -2\sqrt{5}/5)$       (c)  $(\sqrt{5}/5, 2\sqrt{5}/5)$       (d)  $(-\sqrt{5}/5, -2\sqrt{5}/5)$   
(e)  $(\sqrt{5}, \sqrt{10})$

11. Find the line of least squares  $y = ax + b$  which best fits the data points  $(-3, -5)$ ,  $(1, 3)$  and  $(2, 5)$ .

- (a)  $y = 5x + 4$       (b)  $y = 2x + 1$       (c)  $y = 3x + 2$       (d)  $y = x + 1$       (e)  $y = 3x$ .

12. Compute  $\frac{\partial f}{\partial x}(0, 1)$  for the function  $f(x, y) = x^2e^{3x+y} + 5ye^y + 2xy$ .

- (a) 4                      (b) 0                      (c) 2                      (d) 5                      (e) 3

13. Consider the function  $f(x, y) = -6x^2 - 7xy - 2y^2$ . Which of the following statements is true?

- (a) The second derivative test is inconclusive.                      (b)  $(0, 0)$  is a saddle point.  
(c)  $(0, 0)$  is a local minimum.    (d)  $(0, 0)$  is a local maximum.  
(e) There are no critical points.



14. (11 pts) A person opens an Individual Retirement Account (IRA) with the initial amount of \$50,000. Then \$6,000 per year is deposited in this IRA in a uniform and continuous manner. Assume that the interest rate is 7.5% compounded continuously.

(4 pts) Model this problem as a Calculus problem by finding a **differential equation** and an **initial condition** describing the amount of money,  $M(t)$ , in the IRA at any time  $t$ .

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15. (11 pts) The Cobb-Douglas production function for a certain product is given by  $f(x, y) = 5x^{\frac{3}{5}}y^{\frac{2}{5}}$  where  $f(x, y)$  is the quantity produced,  $x$  denotes units of labor force and  $y$  denotes units of capital. Assume that each unit of labor costs \$400, each unit of capital costs \$100, and the total budget is \$20,000. Find the amounts of labor and capital which will maximize the company's production while keeping within the constraints of the budget. (**Remark:** No second derivative test required).

Answer:

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Mult. Choice \_\_\_\_\_  
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15 \_\_\_\_\_  
Total \_\_\_\_\_

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- (a) 9      (b) 3      (c) 0      (d) 10      (e) 12



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- (a)  $y = e^{t-y} + C$       (b)  $y = e^t + C$       (c)  $y = e^{t-y} + e^t + C$       (d)  $y = \ln(e^t + C)$   
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Use linear approximation to estimate the value of  $f(1.1, -3.1)$ .

- (a) 1.5      (b) 2.3      (c) 1.7      (d) 2.1      (e) 1.9

9. Find the only critical point of the function

$$f(x, y) = 3x^2 + y^2 + 2xy - 8x + 4y + 1.$$

- (a)  $(0, 0)$       (b)  $(4, -6)$       (c)  $(4, -8)$       (d)  $(0, 4)$       (e)  $(3, -5)$

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12. Compute  $\frac{\partial f}{\partial x}(0, 1)$  for the function  $f(x, y) = x^2e^{3x+y} + 5ye^y + 2xy$ .

- (a) 0                      (b) 4                      (c) 3                      (d) 2                      (e) 5

13. Consider the function  $f(x, y) = -6x^2 - 7xy - 2y^2$ . Which of the following statements is true?

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Answer:

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3.  a  b  c  d  e

4.  a  b  c  d  e

5.  a  b  c  d  e

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Total \_\_\_\_\_