Name: ______________________ Instructor: ______________________

Math 108 Exam 3

November 21, 2002

This exam consists of a total of 15 questions. There are thirteen multiple choice questions, worth six points each. There are two partial credit problems worth eleven points each. To record your answers to the multiple choice questions, place an **X** through the appropriate box on this page. You are allowed to use calculators on this exam, but you are taking it under the **honor code** as described in the Du-Lac.

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e
4.	a	b	c	d	e
5.	a	b	c	d	e
6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e
11.	a	b	c	d	e
12.	a	b	c	d	e
13.	a	b	c	d	e

Multiple choice ______________

14 ______________

15 ______________

Total score ______________

Multiple Choice Problems (6 Points Each).

You must mark the correct answer on the cover sheet to receive credit!

1. Using Euler's method with $n = 2$, estimate $y(2)$ where

$$\frac{dy}{dt} = e^{y^2-9t}, \quad y(1) = 3.$$

(a) $e^{-1.25}$ (b) 3.5 (c) $3 + e^9$ (d) $3.5 + 0.5e^{-1.25}$ (e) $\frac{1}{9}e^{-4}$

2. Which of the following is the second order Taylor polynomial approximating $f(x) = \ln(2x + 1)$ at $x = 0$?

(a) $x - x^2$ (b) $2x - 2x^2$ (c) $2x - 4x^2$ (d) $1 + x + \dfrac{x^2}{x!}$

(e) $\ln 3 + \dfrac{2}{3}x - \dfrac{4}{18}x^2$

3. Use a Taylor polynomial of degree 3 about $x = 1$ to approximate

$$\int_1^2 \ln(x)\, dx$$

(a) $\frac{5}{12}$ (b) $\frac{23}{24}$ (c) $\frac{9}{24}$ (d) $\frac{19}{12}$ (e) $2\ln 2$

4. Consider the series

$$\sum_{k=0}^{\infty} \left(\frac{5}{4}\right)^k .$$

Which of the following statements is true?

(a) This series converges to 0.
(b) This series converges to -4.
(c) This series converges to 5.
(d) This series converges to $\frac{5}{4}$.
(e) This series diverges.

5. The test for a particular disease is positive 96% of the time when the disease is present. The test is also (false) positive 2% of the time when the disease is not present. If 0.2% of the population has the disease, what is the probability that a person who tests positive actually has the disease? (choose the closest answer).

(a) 0.0019 (b) 0.9620 (c) 0.9600 (d) 0.0400 (e) 0.0878

6. A fair coin is flipped 4 times. What is the probability that it comes up "heads" exactly two times?

(a) $\frac{1}{2}$ (b) $\frac{11}{16}$ (c) $\frac{1}{8}$ (d) $\frac{3}{8}$ (e) $\frac{7}{8}$

7. If E and F are two events satisfying $P(E') = 0.6$, $P(F) = 0.6$ and $P(E \cup F) = 0.7$, find $P(E \cap F)$.

(a) 0.24 (b) 0.3 (c) 0.36 (d) 0.5 (e) 1.0

8. Suppose two cards are drawn in succession from a standard deck. What is the probability that both are of the same suit?

(a) $\frac{12}{51}$ (b) $\frac{1}{16}$ (c) $\frac{1}{4}$ (d) $\frac{48}{51}$ (e) $\frac{156}{2652}$

9. A monkey is randomly hitting keys on a typewriter with 26 keys (i.e., 'A', 'B',...,'Z'). If he types exactly 7 characters independently, what is the probability that it will read "OTHELLO"?

(a) $\left(\frac{5}{26}\right)$ (b) $\left(\frac{7}{26}\right)^7$ (c) $\left(\frac{5}{26}\right)^7$ (d) $\left(\frac{1}{26}\right)^7$ (e) $\left(\frac{1}{26}\right)^5$

10. What is the value of the following series?

$$1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + \left(\frac{3}{5}\right)^3 + \cdots$$

(a) $\frac{5}{3}$ (b) $\frac{5}{2}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$ (e) ∞

11. A $\frac{3}{4}$ pint cup of coffee is sitting on my desk evaporating. Suppose that 1/3 of a pint evaporates on the first day, 1/9 of a pint evaporates on the second day, 1/27 of a pint evaporates on the third day, and so on. If this process continued forever in this way, how much coffee would eventually remain in my cup?

(a) $\frac{1}{4}$ pint (b) $\frac{1}{3}$ pint (c) $\frac{1}{2}$ pint (d) $\frac{3}{2}$ pint (e) None.

12. Consider events E and F satisfying $P(E|F) = 0.3$, $P(E) = 0.1$, $P(F) = 0.6$. What is $P(E \cap F)$?

(a) 0.03 (b) 0.70 (c) 0.18 (d) 0.06

(e) Not enough given information to determine.

13. Which of the following is the Taylor series approximating e^{x+2} about $x = 1$?

(a) $\sum_{k=0}^{\infty} e^3(x-1)^k$ (b) $e^2 \sum_{k=0}^{\infty} \frac{(x-1)^k}{k!}$ (c) $e^3 \sum_{k=0}^{\infty} \frac{x^k}{k!}$ (d) $\sum_{k=0}^{\infty} \frac{(x-1)^k}{k!}$

(e) $e^3 \sum_{k=0}^{\infty} \frac{(x-1)^k}{k!}$

Partial Credit Problems (11 Points Each).

You must show your work to receive credit!

14. (Two parts)

(i) Use Euler's method with $n = 2$ to approximate $y(1)$ where

$$\frac{dy}{dt} = -2y, \qquad y(0) = 1.$$

(ii) Here is the direction field corresponding to the differential equation from part (i). Use it to sketch the solution to the initial value problem in part (1). (you may sketch it right over the direction field).

15. An experiment consists of rolling two dice and observing the numbers on the sides facing up.

(a) List the sample space for this experiment.

(b) Find the probability that the smaller of the two numbers is at least 4.