1. Compute the average value of $f(x) = x^{-1/5} + e^{x/3}$ over the interval $[0, 3]$.

(a) $\frac{5}{12}3^{-6/5} + 3e - 3$ (b) $\frac{5}{12}3^{4/5} + 3e$ (c) $\frac{5}{6}3^{-6/5} + 3e - 3$ (d) $\frac{5}{4}3^{4/5} + 3e - 3$

(e) $\frac{5}{12}3^{4/5} + e - 1$

2. Estimate the integral $\int_0^1 \sqrt{x+2}\ dx$ by dividing the interval [0,1] into five equal segments and using the **Trapezoid Rule**.

(a) $\frac{1}{5}\left(\sqrt{2.0} + 2\sqrt{2.2} + 2\sqrt{2.4} + 2\sqrt{2.6} + 2\sqrt{2.8} + \sqrt{3.0}\right)$

(b) $\frac{1}{10}\left(\sqrt{2.0} + \sqrt{2.25} + \sqrt{2.5} + \sqrt{2.75} + \sqrt{3.0}\right)$

(c) $\frac{1}{5}\left(\sqrt{2.1} + \sqrt{2.3} + \sqrt{2.5} + \sqrt{2.7} + \sqrt{2.9}\right)$

(d) $\frac{1}{10}\left(\sqrt{2.0} + 2\sqrt{2.2} + 2\sqrt{2.4} + 2\sqrt{2.6} + 2\sqrt{2.8} + \sqrt{3.0}\right)$

(e) none of the above

3. Compute the following indefinite integral $\displaystyle\int x^2(3x^3-4)^{5/2}\,dx$.

(a) $\dfrac{18}{7}(3x^3-4)^{7/2}+C$ (b) $\dfrac{x^3}{3}(3x^3-4)^{7/2}+C$ (c) $\dfrac{7}{18}(3x^3-4)^{7/2}+C$

(d) $\dfrac{2x^3}{21}(3x^3-4)^{7/2}+C$ (e) $\dfrac{2}{63}(3x^3-4)^{7/2}+C$

4. Compute the following definite integral $\displaystyle\int_4^6 \frac{1}{x^2+x-2}\,dx$.

(a) $2\ln\dfrac{15}{6}$ (b) $\dfrac{1}{3}\ln\dfrac{2}{15}$ (c) $2\ln\dfrac{5}{6}$ (d) $\dfrac{1}{3}\ln\dfrac{4}{5}$ (e) $\dfrac{1}{3}\ln\dfrac{5}{4}$

5. The demand function for popsicles is $D(q) = \dfrac{30}{q+1}$ and the supply function is $S(q) = q + 2$ (in millions of units and thousands of dollars). Compute the **producer surplus** for popsicles.

(a) 4 (b) $30 \ln 5 - 24$ (c) 8 (d) $30 \ln 5$ (e) 6

6. Yolanda decides to plan for retirement. She thinks she can invest at a continuous rate of 2000 dolars per year. If she is planning on 3% interest (compounded continuously) and wants to retire in 28 years, how would she calculate value of her investments at the end of that period?

(a) $\int_0^{28} (2000)e^{.03t}\,dt$ (b) $\int_0^{28} (2000)e^{.03(t-28)}\,dt$ (c) $\int_0^{28} (2000)e^{.03(28-t)}\,dt$

(d) $e^{(.03\cdot 28)} \int_0^{28} (2000)\,dt$ (e) $\int_0^{28} (2000)e^{-.03t}\,dt$

7. Evaluate the following integral: $\int_1^2 \ln(x)dx$

(a) $2\ln(2) - 1$ (b) $2\ln(2) - \frac{1}{2}$ (c) $\ln(2) + 1$ (d) $\ln(2) - 2$ (e) $-\frac{1}{2}$

8. Solve the initial value problem $y' = 2xe^{x^2}$, $y(0) = 1$ and compute $y(2)$.

(a) $2e^4$ (b) $e^2 + 1$ (c) $e^4 + 1$ (d) e^4 (e) $2e^2 + 1$

9. Solve the following differential equation with given initial condition:

$$\frac{dy}{dt} = ty + t,\ y(0) = 1.$$

Then find $y(1)$.

(a) $2e - 1$ (b) $2e + 1$ (c) $2\sqrt{e} - 1$ (d) $e + 1$ (e) $\sqrt{e} - 1$

10. A person wants to take out a 30-year mortgage to buy a house. The interest rate for this mortgage is 8%, and the payments are $24,000 per year. Let $M(t)$ be the amount of money owed in year t . Model this situation as an initial value problem.

(a) $M'(t) = 24,000M - 0.08,\ M(0) = 30$ (b) $M'(t) = 0.08M - 24,000,\ M(30) = 24,000$

(c) $M'(t) = 0.08M + 24,000,\ M(30) = 0$ (d) $M'(t) = 0.08M,\ M(0) = 24,000$

(e) $M'(t) = 0.08M - 24,000,\ M(30) = 0$

11. The population of a colony of bacteria follows the logistic growth model. The colony has intrinsic growth rate $r = 0.04$. After a very long time, the population stabilizes at 10 billion. Let $p(t)$ be the population of the colony (measured in billions) at time t . Which of the following differential equations models this situation?

(a) $\dfrac{dp}{dt} = 0.04p(1 - 0.004p)$ (b) $\dfrac{dp}{dt} = 0.004p - 0.04p^2$ (c) $\dfrac{dp}{dt} = 0.04p - 0.004p^2$

(d) $\dfrac{dp}{dt} = 0.04p(1 - 10p)$ (e) $\dfrac{dp}{dt} = 0.04p - 10p^2$

12. Compute the improper integral

$$\int_{-\infty}^{\infty} 4xe^{-x^2}\,dx.$$

(a) $-\infty$ (b) 0 (c) 2 (d) $+\infty$ (e) -2

13. Let

$$f(x, y) = 3xe^y + x^2y^2.$$

Find the equation of the tangent plane to the graph at the point $(1, 0, 3)$. Which of the following points lies on this plane?

(a) $(0, 1, 5)$ (b) $(0, 1, 1)$ (c) $(0, 1, 3)$ (d) $(0, 1, 0)$ (e) $(0, 1, 6)$

14. Consider the function $f(x, y) = x^3 + y^3 - 3x - 3y$. Which of the following is a critical point?

(a) $(3, 3)$ (b) $(-1, 1)$ (c) $(0, 0)$ (d) $(3, -3)$ (e) $(\sqrt{3}, -\sqrt{3})$

15. The following high temperatures (in degrees Celsius) were recorded in South Bend, Indiana:

Date	Temperature
January 3	3
January 4	1
January 5	-2

Find the line of least squares $y = ax + b$ which best fits these data points, where x is the date in January and y is the high temperature for that day. Predict the high temperature on January 6 (rounded to the nearest tenth of a degree).

(a) -4.5 (b) -5.2 (c) -4.7 (d) -4.3 (e) -4.1

16. A company makes two products X and Y. If x is the quantity of X produced in a month and y is the quantity of Y produced in a month, then the total profit is given by the function $P(x, y) = 30x + 40y - 0.2x^2 - 0.2y^2$. On the other hand, the size of the company's factory limits the possible values of x and y by the equation $x + y = 125$. Use Lagrange multipliers to find the values of x and y that maximize profit.

(a) $x = 75, y = 50$ (b) $x = 125, y = 0$ (c) $x = 0, y = 125$ (d) $x = 62.5, y = 62.5$

(e) $x = 50, y = 75$

17. Using Euler's method with $n = 2$, estimate $y(1)$ where

$$\frac{dy}{dt} = y + e^{-t^2}, \qquad y(0) = 5.$$

(a) $12 + 0.5e^{-0.25}$. (b) $0.5(5 + e^{-.25})$. (c) 5.5 (d) $10.5 + e^{-0.25}$ (e) $5 + e^{-0.5}$

18. Which of the following is the second order Taylor polynomial approximating $f(x) = \ln(2x+1)$ at $x = 0$?

(a) $2x - 4x^2$ (b) $1 + x + \dfrac{x^2}{x!}$ (c) $x - x^2$ (d) $\ln 3 + \dfrac{2}{3}x - \dfrac{4}{18}x^2$ (e) $2x - 2x^2$

19. Use a Taylor polynomial of degree 2 about $x = 0$ to approximate

$$\int_{-1}^{1} \sqrt{1 - x^2}\, dx$$

(a) $\frac{17}{10}$ (b) $\frac{31}{20}$ (c) $\frac{5}{3}$ (d) 0 (e) $\frac{7}{3}$

20. The test for a particular disease is positive 98% of the time when the disease is present. The test is also (false) positive 1% of the time when the disease is not present. If 0.05% of the population has the disease, what is the probability that a person who tests positive actually has the disease? (choose the closest answer).

(a) 0.047 (b) 0.980 (c) 0.953 (d) 0.067 (e) 0.015

21. A fair coin is flipped 4 times. What is the probability that it comes up "tails" at least two times?

(a) $\frac{2}{16}$ (b) $\frac{10}{16}$ (c) $\frac{6}{16}$ (d) $\frac{7}{8}$ (e) $\frac{11}{16}$

22. If E and F are two events in the same sample space satisfying $P(E) = 0.4$, $P(E \cup F) = 0.6$, and $P(E \cap F) = 0.1$, find $P(F)$.

(a) 0.2 (b) 0.25 (c) 0.6 (d) 0.3 (e) 0.04

23. What is the value of the following series?

$$1 + \frac{5}{7} + \left(\frac{5}{7}\right)^2 + \left(\frac{5}{7}\right)^3 + \cdots$$

(a) $\frac{7}{2}$ (b) $\frac{2}{5}$ (c) $\frac{7}{5}$ (d) None of these - it diverges. (e) $\frac{2}{7}$

24. Which of the following is the Taylor series of e^x about $x = 1$?

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{ex^k}{k!}$ (b) $\displaystyle\sum_{k=1}^{\infty} \frac{x^k}{k!}$ (c) $\displaystyle\sum_{k=0}^{\infty} \frac{e(x-1)^k}{k!}$ (d) $\displaystyle\sum_{k=1}^{\infty} \frac{(x-1)}{k!}$ (e) $\displaystyle\sum_{k=0}^{\infty} \frac{(x-1)^k}{k!}$

25. A pair of dice is rolled, and the sum is known to be 10 or more. What is the probability that the sum is 12?

(a) $\frac{1}{36}$ (b) $\frac{1}{3}$ (c) $\frac{1}{9}$ (d) $\frac{5}{36}$ (e) $\frac{1}{6}$

26. A continuous random variable X has a probability density function $f(x) = \frac{3}{8}x^2$ for $0 \leq x \leq 2$. What is the expected value $E(X)$?

(a) 1 (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{3}{16}$ (e) $\frac{4}{3}$

27. Let X be a discrete random variable whose distribution is given by

x	2	4	6	8	10	12	14
$f(x)$	0	$\frac{1}{5}$	0	0	$\frac{1}{5}$	$\frac{3}{5}$	0

Compute the variance Var (X) .

(a) $\frac{48}{5}$ (b) $\frac{12}{5}$ (c) $\frac{27}{5}$ (d) $\frac{36}{5}$ (e) $\frac{32}{5}$

28. A fair coin is flipped 10 times. What is the probability that at least 9 heads do appear?

(a) $\frac{1}{256}$ (b) $\frac{1}{128}$ (c) $\frac{5}{512}$ (d) $\frac{11}{1024}$ (e) $\frac{9}{1024}$

29. The length of time that a truck's brake pads last (measured in years) is a random variable that is exponentially distributed with expectation 2 . What is the probability that the brake pads will last less than 6 months?

(a) 0.06 (b) 0.22 (c) 0.95 (d) 0.14 (e) 0.63

30. A normal random variable X has mean $\mu = 8$ and standard deviation $\sigma = 4$. Calculate $P(7.36 \leq X \leq 9.76)$. You will need the attached table of values for the function $\Phi(x)$.

(a) 0.6700 (b) 0.1064 (c) 0.5636 (d) 0.2336 (e) 0.0655

Version 1 color:

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Math 108, Final Exam

December 17, 2002

1. Please cross ☒ the correct answers.
2. This test will be exactly 120 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 17 PAGES with 30 PROBLEMS. Each problem has an equal point value of 5 points. Use the back of the test pages for scratch work.

Name: ______________________________

Prof: ______________________________

1.	a	b	c	d	•
2.	a	b	c	•	e
3.	a	b	c	d	•
4.	a	b	c	d	•
5.	a	b	•	d	e
6.	a	b	•	d	e
7.	•	b	c	d	e
8.	a	b	c	•	e
9.	a	b	•	d	e
10.	a	b	c	d	•
11.	a	b	•	d	e
12.	a	•	c	d	e
13.	a	b	•	d	e
14.	a	•	c	d	e
15.	a	b	c	•	e
16.	a	b	c	d	•
17.	•	b	c	d	e
18.	a	b	c	d	•
	5a's	3b's	8c's	6d's	8e's

19.	a	b	•	d	e
20.	•	b	c	d	e
21.	a	b	c	d	•
22.	a	b	c	•	e
23.	•	b	c	d	e
24.	a	b	•	d	e
25.	a	b	c	d	•
26.	a	b	•	d	e
27.	•	b	c	d	e
28.	a	b	c	•	e
29.	a	•	c	d	e
30.	a	b	c	•	e

Total ________

Math 108, Final Exam

December 17, 2002

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Name: ____________________

Prof: ____________________

1.	a	b	c	d	e
2.	a	b	c	d	e
3.	a	b	c	d	e
4.	a	b	c	d	e
5.	a	b	c	d	e
6.	a	b	c	d	e
7.	a	b	c	d	e
8.	a	b	c	d	e
9.	a	b	c	d	e
10.	a	b	c	d	e
11.	a	b	c	d	e
12.	a	b	c	d	e
13.	a	b	c	d	e
14.	a	b	c	d	e
15.	a	b	c	d	e
16.	a	b	c	d	e
17.	a	b	c	d	e
18.	a	b	c	d	e
19.	a	b	c	d	e
20.	a	b	c	d	e
21.	a	b	c	d	e
22.	a	b	c	d	e
23.	a	b	c	d	e
24.	a	b	c	d	e
25.	a	b	c	d	e
26.	a	b	c	d	e
27.	a	b	c	d	e
28.	a	b	c	d	e
29.	a	b	c	d	e
30.	a	b	c	d	e

Total ________

Sign your name:

1. Compute the average value of $f(x) = x^{-1/5} + e^{x/3}$ over the interval $[0, 3]$.

(a) $\frac{5}{12}3^{4/5} + 3e$ (b) $\frac{5}{6}3^{-6/5} + 3e - 3$ (c) $\frac{5}{4}3^{4/5} + 3e - 3$ (d) $\frac{5}{12}3^{4/5} + e - 1$

(e) $\frac{5}{12}3^{-6/5} + 3e - 3$

2. Estimate the integral $\int_0^1 \sqrt{x+2}\, dx$ by dividing the interval [0,1] into five equal segments and using the **Trapezoid Rule**.

(a) $\frac{1}{10}\left(\sqrt{2.0} + \sqrt{2.25} + \sqrt{2.5} + \sqrt{2.75} + \sqrt{3.0}\right)$

(b) $\frac{1}{5}\left(\sqrt{2.0} + 2\sqrt{2.2} + 2\sqrt{2.4} + 2\sqrt{2.6} + 2\sqrt{2.8} + \sqrt{3.0}\right)$

(c) $\frac{1}{5}\left(\sqrt{2.1} + \sqrt{2.3} + \sqrt{2.5} + \sqrt{2.7} + \sqrt{2.9}\right)$

(d) $\frac{1}{10}\left(\sqrt{2.0} + 2\sqrt{2.2} + 2\sqrt{2.4} + 2\sqrt{2.6} + 2\sqrt{2.8} + \sqrt{3.0}\right)$

(e) none of the above

3. Compute the following indefinite integral $\int x^2(3x^3-4)^{5/2}\,dx$.

(a) $\frac{2x^3}{21}(3x^3-4)^{7/2}+C$ (b) $\frac{x^3}{3}(3x^3-4)^{7/2}+C$ (c) $\frac{2}{63}(3x^3-4)^{7/2}+C$

(d) $\frac{18}{7}(3x^3-4)^{7/2}+C$ (e) $\frac{7}{18}(3x^3-4)^{7/2}+C$

4. Compute the following definite integral $\int_4^6 \frac{1}{x^2+x-2}\,dx$.

(a) $2\ln\frac{15}{6}$ (b) $2\ln\frac{5}{6}$ (c) $\frac{1}{3}\ln\frac{2}{15}$ (d) $\frac{1}{3}\ln\frac{4}{5}$ (e) $\frac{1}{3}\ln\frac{5}{4}$

5. The demand function for popsicles is $D(q) = \dfrac{30}{q+1}$ and the supply function is $S(q) = q + 2$ (in millions of units and thousands of dollars). Compute the **producer surplus** for popsicles.

(a) 6 (b) $30 \ln 5 - 24$ (c) $30 \ln 5$ (d) 8 (e) 4

6. Yolanda decides to plan for retirement. She thinks she can invest at a continuous rate of 2000 dolars per year. If she is planning on 3% interest (compounded continuously) and wants to retire in 28 years, how would she calculate value of her investments at the end of that period?

(a) $\int_0^{28} (2000)e^{-.03t}dt$ (b) $\int_0^{28} (2000)e^{.03(t-28)}dt$ (c) $\int_0^{28} (2000)e^{.03t}dt$

(d) $\int_0^{28} (2000)e^{.03(28-t)}dt$ (e) $e^{(.03 \cdot 28)} \int_0^{28} (2000)dt$

7. Evaluate the following integral: $\int_1^2 \ln(x)dx$

(a) $\ln(2) + 1$ (b) $\ln(2) - 2$ (c) $2\ln(2) - \frac{1}{2}$ (d) $2\ln(2) - 1$ (e) $-\frac{1}{2}$

8. Solve the initial value problem $y' = 2xe^{x^2}$, $y(0) = 1$ and compute $y(2)$.

(a) $2e^4$ (b) $2e^2 + 1$ (c) $e^2 + 1$ (d) $e^4 + 1$ (e) e^4

9. Solve the following differential equation with given initial condition:

$$\frac{dy}{dt} = ty + t,\ y(0) = 1.$$

Then find $y(1)$.

(a) $2e - 1$ (b) $2e + 1$ (c) $\sqrt{e} - 1$ (d) $e + 1$ (e) $2\sqrt{e} - 1$

10. A person wants to take out a 30-year mortgage to buy a house. The interest rate for this mortgage is 8%, and the payments are $24,000 per year. Let $M(t)$ be the amount of money owed in year t . Model this situation as an initial value problem.

(a) $M'(t) = 0.08M - 24,000,\ M(30) = 0$ (b) $M'(t) = 0.08M + 24,000,\ M(30) = 0$
(c) $M'(t) = 0.08M - 24,000,\ M(30) = 24,000$ (d) $M'(t) = 24,000M - 0.08,\ M(0) = 30$

(e) $M'(t) = 0.08M,\ M(0) = 24,000$

11. The population of a colony of bacteria follows the logistic growth model. The colony has intrinsic growth rate $r = 0.04$. After a very long time, the population stabilizes at 10 billion. Let $p(t)$ be the population of the colony (measured in billions) at time t . Which of the following differential equations models this situation?

(a) $\dfrac{dp}{dt} = 0.04p - 10p^2$ (b) $\dfrac{dp}{dt} = 0.04p - 0.004p^2$ (c) $\dfrac{dp}{dt} = 0.04p(1 - 10p)$

(d) $\dfrac{dp}{dt} = 0.004p - 0.04p^2$ (e) $\dfrac{dp}{dt} = 0.04p(1 - 0.004p)$

12. Compute the improper integral

$$\int_{-\infty}^{\infty} 4xe^{-x^2}\, dx.$$

(a) -2 (b) 2 (c) $-\infty$ (d) $+\infty$ (e) 0

13. Let

$$f(x, y) = 3xe^y + x^2y^2.$$

Find the equation of the tangent plane to the graph at the point $(1, 0, 3)$. Which of the following points lies on this plane?

(a) $(0, 1, 5)$ (b) $(0, 1, 0)$ (c) $(0, 1, 6)$ (d) $(0, 1, 3)$ (e) $(0, 1, 1)$

14. Consider the function $f(x, y) = x^3 + y^3 - 3x - 3y$. Which of the following is a critical point?

(a) $(3, 3)$ (b) $(0, 0)$ (c) $(-1, 1)$ (d) $(3, -3)$ (e) $(\sqrt{3}, -\sqrt{3})$

15. The following high temperatures (in degrees Celsius) were recorded in South Bend, Indiana:

Date	Temperature
January 3	3
January 4	1
January 5	-2

Find the line of least squares $y = ax + b$ which best fits these data points, where x is the date in January and y is the high temperature for that day. Predict the high temperature on January 6 (rounded to the nearest tenth of a degree).

(a) -4.1 (b) -4.3 (c) -5.2 (d) -4.5 (e) -4.7

16. A company makes two products X and Y. If x is the quantity of X produced in a month and y is the quantity of Y produced in a month, then the total profit is given by the function $P(x, y) = 30x + 40y - 0.2x^2 - 0.2y^2$. On the other hand, the size of the company's factory limits the possible values of x and y by the equation $x + y = 125$. Use Lagrange multipliers to find the values of x and y that maximize profit.

(a) $x = 75, y = 50$ (b) $x = 0, y = 125$ (c) $x = 62.5, y = 62.5$ (d) $x = 50, y = 75$

(e) $x = 125, y = 0$

17. Using Euler's method with $n = 2$, estimate $y(1)$ where

$$\frac{dy}{dt} = y + e^{-t^2}, \qquad y(0) = 5.$$

(a) $5 + e^{-0.5}$ (b) $12 + 0.5e^{-0.25}$. (c) $10.5 + e^{-0.25}$ (d) 5.5 (e) $0.5(5 + e^{-.25})$.

18. Which of the following is the second order Taylor polynomial approximating $f(x) = \ln(2x+1)$ at $x = 0$?

(a) $\ln 3 + \frac{2}{3}x - \frac{4}{18}x^2$ (b) $2x - 4x^2$ (c) $x - x^2$ (d) $1 + x + \frac{x^2}{x!}$ (e) $2x - 2x^2$

19. Use a Taylor polynomial of degree 2 about $x = 0$ to approximate

$$\int_{-1}^{1} \sqrt{1 - x^2}\, dx$$

(a) $\frac{5}{3}$ (b) 0 (c) $\frac{17}{10}$ (d) $\frac{31}{20}$ (e) $\frac{7}{3}$

20. The test for a particular disease is positive 98% of the time when the disease is present. The test is also (false) positive 1% of the time when the disease is not present. If 0.05% of the population has the disease, what is the probability that a person who tests positive actually has the disease? (choose the closest answer).

(a) 0.067 (b) 0.047 (c) 0.015 (d) 0.980 (e) 0.953

21. A fair coin is flipped 4 times. What is the probability that it comes up "tails" at least two times?

(a) $\frac{11}{16}$ (b) $\frac{7}{8}$ (c) $\frac{2}{16}$ (d) $\frac{10}{16}$ (e) $\frac{6}{16}$

22. If E and F are two events in the same sample space satisfying $P(E) = 0.4$, $P(E \cup F) = 0.6$, and $P(E \cap F) = 0.1$, find $P(F)$.

(a) 0.2 (b) 0.04 (c) 0.25 (d) 0.3 (e) 0.6

23. What is the value of the following series?

$$1 + \frac{5}{7} + \left(\frac{5}{7}\right)^2 + \left(\frac{5}{7}\right)^3 + \cdots$$

(a) $\frac{2}{7}$ (b) None of these - it diverges. (c) $\frac{7}{2}$ (d) $\frac{2}{5}$ (e) $\frac{7}{5}$

24. Which of the following is the Taylor series of e^x about $x = 1$?

(a) $\sum_{k=0}^{\infty} \frac{ex^k}{k!}$ (b) $\sum_{k=1}^{\infty} \frac{x^k}{k!}$ (c) $\sum_{k=0}^{\infty} \frac{(x-1)^k}{k!}$ (d) $\sum_{k=1}^{\infty} \frac{(x-1)}{k!}$ (e) $\sum_{k=0}^{\infty} \frac{e(x-1)^k}{k!}$

25. A pair of dice is rolled, and the sum is known to be 10 or more. What is the probability that the sum is 12?

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{5}{36}$ (d) $\frac{1}{9}$ (e) $\frac{1}{36}$

26. A continuous random variable X has a probability density function $f(x) = \frac{3}{8}x^2$ for $0 \leq x \leq 2$. What is the expected value $E(X)$?

(a) $\frac{3}{16}$ (b) $\frac{5}{2}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$ (e) 1

27. Let X be a discrete random variable whose distribution is given by

x	2	4	6	8	10	12	14
$f(x)$	0	$\frac{1}{5}$	0	0	$\frac{1}{5}$	$\frac{3}{5}$	0

Compute the variance Var (X) .

(a) $\frac{12}{5}$ (b) $\frac{32}{5}$ (c) $\frac{48}{5}$ (d) $\frac{36}{5}$ (e) $\frac{27}{5}$

28. A fair coin is flipped 10 times. What is the probability that at least 9 heads do appear?

(a) $\frac{5}{512}$ (b) $\frac{9}{1024}$ (c) $\frac{1}{128}$ (d) $\frac{1}{256}$ (e) $\frac{11}{1024}$

29. The length of time that a truck's brake pads last (measured in years) is a random variable that is exponentially distributed with expectation 2 . What is the probability that the brake pads will last less than 6 months?

(a) 0.22 (b) 0.95 (c) 0.06 (d) 0.14 (e) 0.63

30. A normal random variable X has mean $\mu = 8$ and standard deviation $\sigma = 4$. Calculate $P(7.36 \leq X \leq 9.76)$. You will need the attached table of values for the function $\Phi(x)$.

(a) 0.1064 (b) 0.5636 (c) 0.2336 (d) 0.0655 (e) 0.6700

Version 2 color:

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Math 108, Final Exam

December 17, 2002

1. Please cross [×] the correct answers.
2. This test will be exactly 120 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 17 PAGES with 30 PROBLEMS. Each problem has an equal point value of 5 points. Use the back of the test pages for scratch work.

Name: ______________________________

Prof: ______________________________

1.	a	b	c	•	e
2.	a	b	c	•	e
3.	a	b	•	d	e
4.	a	b	c	d	•
5.	a	b	c	•	e
6.	a	b	c	•	e
7.	a	b	c	•	e
8.	a	b	c	d	•
9.	a	b	c	d	•
10.	•	b	c	d	e
11.	a	•	c	d	e
12.	a	b	c	d	•
13.	a	b	c	•	e
14.	a	b	•	d	e
15.	a	•	c	d	e
16.	a	b	c	•	e
17.	a	•	c	d	e
18.	a	b	c	d	•

4a's 5b's 6c's 8d's 7e's

19.	•	b	c	d	e
20.	a	•	c	d	e
21.	•	b	c	d	e
22.	a	b	c	•	e
23.	a	b	•	d	e
24.	a	b	c	d	•
25.	a	•	c	d	e
26.	a	b	•	d	e
27.	a	b	•	d	e
28.	a	b	c	d	•
29.	•	b	c	d	e
30.	a	b	•	d	e

Total ________

1. Compute the average value of $f(x) = x^{-1/5} + e^{x/3}$ over the interval $[0, 3]$.

(a) $\frac{5}{4}3^{4/5} + 3e - 3$ (b) $\frac{5}{12}3^{-6/5} + 3e - 3$ (c) $\frac{5}{6}3^{-6/5} + 3e - 3$ (d) $\frac{5}{12}3^{4/5} + e - 1$

(e) $\frac{5}{12}3^{4/5} + 3e$

2. Estimate the integral $\int_0^1 \sqrt{x+2}\, dx$ by dividing the interval [0,1] into five equal segments and using the **Trapezoid Rule**.

(a) $\frac{1}{5}\left(\sqrt{2.0} + 2\sqrt{2.2} + 2\sqrt{2.4} + 2\sqrt{2.6} + 2\sqrt{2.8} + \sqrt{3.0}\right)$

(b) $\frac{1}{10}\left(\sqrt{2.0} + 2\sqrt{2.2} + 2\sqrt{2.4} + 2\sqrt{2.6} + 2\sqrt{2.8} + \sqrt{3.0}\right)$

(c) $\frac{1}{10}\left(\sqrt{2.0} + \sqrt{2.25} + \sqrt{2.5} + \sqrt{2.75} + \sqrt{3.0}\right)$

(d) $\frac{1}{5}\left(\sqrt{2.1} + \sqrt{2.3} + \sqrt{2.5} + \sqrt{2.7} + \sqrt{2.9}\right)$

(e) none of the above

3. Compute the following indefinite integral $\int x^2(3x^3-4)^{5/2}\,dx$.

(a) $\frac{7}{18}(3x^3-4)^{7/2}+C$ (b) $\frac{2}{63}(3x^3-4)^{7/2}+C$ (c) $\frac{18}{7}(3x^3-4)^{7/2}+C$

(d) $\frac{2x^3}{21}(3x^3-4)^{7/2}+C$ (e) $\frac{x^3}{3}(3x^3-4)^{7/2}+C$

4. Compute the following definite integral $\int_4^6 \frac{1}{x^2+x-2}\,dx$.

(a) $2\ln\frac{15}{6}$ (b) $\frac{1}{3}\ln\frac{5}{4}$ (c) $\frac{1}{3}\ln\frac{4}{5}$ (d) $2\ln\frac{5}{6}$ (e) $\frac{1}{3}\ln\frac{2}{15}$

5. The demand function for popsicles is $D(q) = \dfrac{30}{q+1}$ and the supply function is $S(q) = q + 2$ (in millions of units and thousands of dollars). Compute the **producer surplus** for popsicles.

(a) $30 \ln 5$ (b) 8 (c) 6 (d) $30 \ln 5 - 24$ (e) 4

6. Yolanda decides to plan for retirement. She thinks she can invest at a continuous rate of 2000 dolars per year. If she is planning on 3% interest (compounded continuously) and wants to retire in 28 years, how would she calculate value of her investments at the end of that period?

(a) $\int_0^{28} (2000)e^{.03(t-28)}\,dt$ (b) $\int_0^{28} (2000)e^{.03t}\,dt$ (c) $\int_0^{28} (2000)e^{.03(28-t)}\,dt$

(d) $e^{(.03 \cdot 28)} \int_0^{28} (2000)\,dt$ (e) $\int_0^{28} (2000)e^{-.03t}\,dt$

7. Evaluate the following integral: $\int_1^2 \ln(x)dx$

(a) $2\ln(2) - 1$ (b) $\ln(2) + 1$ (c) $-\frac{1}{2}$ (d) $\ln(2) - 2$ (e) $2\ln(2) - \frac{1}{2}$

8. Solve the initial value problem $y' = 2xe^{x^2}$, $y(0) = 1$ and compute $y(2)$.

(a) $2e^4$ (b) $2e^2 + 1$ (c) e^4 (d) $e^2 + 1$ (e) $e^4 + 1$

9. Solve the following differential equation with given initial condition:

$$\frac{dy}{dt} = ty + t, \; y(0) = 1.$$

Then find $y(1)$.

(a) $2e - 1$ (b) $2e + 1$ (c) $2\sqrt{e} - 1$ (d) $e + 1$ (e) $\sqrt{e} - 1$

10. A person wants to take out a 30-year mortgage to buy a house. The interest rate for this mortgage is 8%, and the payments are $24,000 per year. Let $M(t)$ be the amount of money owed in year t . Model this situation as an initial value problem.

(a) $M'(t) = 0.08M - 24,000,\; M(30) = 0$

(b) $M'(t) = 0.08M,\; M(0) = 24,000$

(c) $M'(t) = 24,000M - 0.08,\; M(0) = 30$

(d) $M'(t) = 0.08M - 24,000,\; M(30) = 24,000$

(e) $M'(t) = 0.08M + 24,000,\; M(30) = 0$

11. The population of a colony of bacteria follows the logistic growth model. The colony has intrinsic growth rate $r = 0.04$. After a very long time, the population stabilizes at 10 billion. Let $p(t)$ be the population of the colony (measured in billions) at time t. Which of the following differential equations models this situation?

(a) $\dfrac{dp}{dt} = 0.04p - 0.004p^2$ (b) $\dfrac{dp}{dt} = 0.04p(1 - 10p)$ (c) $\dfrac{dp}{dt} = 0.04p - 10p^2$

(d) $\dfrac{dp}{dt} = 0.004p - 0.04p^2$ (e) $\dfrac{dp}{dt} = 0.04p(1 - 0.004p)$

12. Compute the improper integral

$$\int_{-\infty}^{\infty} 4xe^{-x^2}\, dx.$$

(a) 0 (b) -2 (c) 2 (d) $+\infty$ (e) $-\infty$

13. Let

$$f(x, y) = 3xe^y + x^2y^2.$$

Find the equation of the tangent plane to the graph at the point $(1, 0, 3)$. Which of the following points lies on this plane?

(a) $(0, 1, 6)$ (b) $(0, 1, 1)$ (c) $(0, 1, 0)$ (d) $(0, 1, 3)$ (e) $(0, 1, 5)$

14. Consider the function $f(x, y) = x^3 + y^3 - 3x - 3y$. Which of the following is a critical point?

(a) $(0, 0)$ (b) $(3, 3)$ (c) $(\sqrt{3}, -\sqrt{3})$ (d) $(3, -3)$ (e) $(-1, 1)$

15. The following high temperatures (in degrees Celsius) were recorded in South Bend, Indiana:

Date	Temperature
January 3	3
January 4	1
January 5	-2

Find the line of least squares $y = ax + b$ which best fits these data points, where x is the date in January and y is the high temperature for that day. Predict the high temperature on January 6 (rounded to the nearest tenth of a degree).

(a) -5.2 (b) -4.1 (c) -4.3 (d) -4.7 (e) -4.5

16. A company makes two products X and Y. If x is the quantity of X produced in a month and y is the quantity of Y produced in a month, then the total profit is given by the function $P(x, y) = 30x + 40y - 0.2x^2 - 0.2y^2$. On the other hand, the size of the company's factory limits the possible values of x and y by the equation $x + y = 125$. Use Lagrange multipliers to find the values of x and y that maximize profit.

(a) $x = 75, y = 50$ (b) $x = 50, y = 75$ (c) $x = 125, y = 0$ (d) $x = 62.5, y = 62.5$

(e) $x = 0, y = 125$

17. Using Euler's method with $n = 2$, estimate $y(1)$ where

$$\frac{dy}{dt} = y + e^{-t^2}, \qquad y(0) = 5.$$

(a) 5.5 (b) $10.5 + e^{-0.25}$ (c) $5 + e^{-0.5}$ (d) $12 + 0.5e^{-0.25}$. (e) $0.5(5 + e^{-.25})$.

18. Which of the following is the second order Taylor polynomial approximating $f(x) = \ln(2x+1)$ at $x = 0$?

(a) $1 + x + \frac{x^2}{x!}$ (b) $2x - 2x^2$ (c) $x - x^2$ (d) $\ln 3 + \frac{2}{3}x - \frac{4}{18}x^2$ (e) $2x - 4x^2$

19. Use a Taylor polynomial of degree 2 about $x = 0$ to approximate

$$\int_{-1}^{1} \sqrt{1 - x^2}\, dx$$

(a) $\frac{5}{3}$ (b) $\frac{31}{20}$ (c) 0 (d) $\frac{17}{10}$ (e) $\frac{7}{3}$

20. The test for a particular disease is positive 98% of the time when the disease is present. The test is also (false) positive 1% of the time when the disease is not present. If 0.05% of the population has the disease, what is the probability that a person who tests positive actually has the disease? (choose the closest answer).

(a) 0.980 (b) 0.953 (c) 0.047 (d) 0.067 (e) 0.015

21. A fair coin is flipped 4 times. What is the probability that it comes up "tails" at least two times?

(a) $\frac{11}{16}$ (b) $\frac{6}{16}$ (c) $\frac{2}{16}$ (d) $\frac{7}{8}$ (e) $\frac{10}{16}$

22. If E and F are two events in the same sample space satisfying $P(E) = 0.4$, $P(E \cup F) = 0.6$, and $P(E \cap F) = 0.1$, find $P(F)$.

(a) 0.2 (b) 0.6 (c) 0.25 (d) 0.3 (e) 0.04

23. What is the value of the following series?

$$1 + \frac{5}{7} + \left(\frac{5}{7}\right)^2 + \left(\frac{5}{7}\right)^3 + \cdots$$

(a) $\frac{7}{5}$ (b) None of these - it diverges. (c) $\frac{7}{2}$ (d) $\frac{2}{5}$ (e) $\frac{2}{7}$

24. Which of the following is the Taylor series of e^x about $x = 1$?

(a) $\sum_{k=1}^{\infty} \frac{x^k}{k!}$ (b) $\sum_{k=0}^{\infty} \frac{(x-1)^k}{k!}$ (c) $\sum_{k=0}^{\infty} \frac{e(x-1)^k}{k!}$ (d) $\sum_{k=0}^{\infty} \frac{ex^k}{k!}$ (e) $\sum_{k=1}^{\infty} \frac{(x-1)}{k!}$

25. A pair of dice is rolled, and the sum is known to be 10 or more. What is the probability that the sum is 12?

(a) $\frac{1}{9}$ (b) $\frac{1}{6}$ (c) $\frac{5}{36}$ (d) $\frac{1}{3}$ (e) $\frac{1}{36}$

26. A continuous random variable X has a probability density function $f(x) = \frac{3}{8}x^2$ for $0 \leq x \leq 2$. What is the expected value $E(X)$?

(a) $\frac{5}{2}$ (b) 1 (c) $\frac{4}{3}$ (d) $\frac{3}{16}$ (e) $\frac{3}{2}$

27. Let X be a discrete random variable whose distribution is given by

x	2	4	6	8	10	12	14
$f(x)$	0	$\frac{1}{5}$	0	0	$\frac{1}{5}$	$\frac{3}{5}$	0

Compute the variance Var (X) .

(a) $\frac{36}{5}$ (b) $\frac{27}{5}$ (c) $\frac{32}{5}$ (d) $\frac{12}{5}$ (e) $\frac{48}{5}$

28. A fair coin is flipped 10 times. What is the probability that at least 9 heads do appear?

(a) $\frac{11}{1024}$ (b) $\frac{1}{256}$ (c) $\frac{5}{512}$ (d) $\frac{9}{1024}$ (e) $\frac{1}{128}$

29. The length of time that a truck's brake pads last (measured in years) is a random variable that is exponentially distributed with expectation 2 . What is the probability that the brake pads will last less than 6 months?

(a) 0.14 (b) 0.22 (c) 0.63 (d) 0.06 (e) 0.95

30. A normal random variable X has mean $\mu = 8$ and standard deviation $\sigma = 4$. Calculate $P(7.36 \leq X \leq 9.76)$. You will need the attached table of values for the function $\Phi(x)$.

(a) 0.2336 (b) 0.1064 (c) 0.6700 (d) 0.5636 (e) 0.0655

Version 3 color:

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Math 108, Final Exam

December 17, 2002

1. Please cross $\boxed{\times}$ the correct answers.
2. This test will be exactly 120 minutes in length. When you are told to begin, but not before, glance through the entire test and put your name on each page. It is YOUR RESPONSIBILITY to make sure your test consists of 17 PAGES with 30 PROBLEMS. Each problem has an equal point value of 5 points. Use the back of the test pages for scratch work.

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6.	a	b	•	d	e
7.	•	b	c	d	e
8.	a	b	•	d	e
9.	a	b	•	d	e
10.	•	b	c	d	e
11.	•	b	c	d	e
12.	•	b	c	d	e
13.	a	b	c	•	e
14.	a	b	c	d	•
15.	a	b	•	d	e
16.	a	•	c	d	e
17.	a	b	c	•	e
18.	a	•	c	d	e

8a's 8b's 7c's 4d's 3e's

19.	•	b	c	d	e
20.	a	b	•	d	e
21.	•	b	c	d	e
22.	a	b	c	•	e
23.	a	b	•	d	e
24.	a	b	•	d	e
25.	a	•	c	d	e
26.	a	b	c	d	•
27.	a	b	c	d	•
28.	•	b	c	d	e
29.	a	•	c	d	e
30.	•	b	c	d	e

Total ________