Math 108 Name:____________________________

Exam 1 Instructor:____________________________

September 26, 2002 Section:____________________________

Calculators are allowed. Do not remove this answer page – you will return the whole exam. You will be allowed 1 hour and 15 minutes to do the test. You may leave earlier if you are finished.

Part I consists of 13 multiple choice questions worth 6 points each. Record your answers by placing an $\times$ through one letter for each problem on this answer sheet.

Part II consists of 2 partial credit problems worth a total of 11 points. Write your answer and show **all** your work on the page on which the question appears.

You are taking this exam under the honor code.

GOOD LUCK

1. a b c d e
2. a b c d e
3. a b c d e
4. a b c d e
5. a b c d e
6. a b c d e
7. a b c d e
8. a b c d e
9. a b c d e
10. a b c d e
11. a b c d e
12. a b c d e
13. a b c d e

For grading use:

1-13	
14	
15	
Total	

Part I: Multiple choice questions (6 points each)

1. Find the solution to the initial value problem $\dfrac{dy}{dt} = \dfrac{4}{t^3}$, $y(1) = 1$.

(a) $y = \dfrac{2}{t^2} + 1$ (b) $y = \dfrac{1}{t^4}$ (c) $y = \dfrac{-1}{t^4} + 2$ (d) $y = \dfrac{16}{t^4} - 15$ (e) $y = \dfrac{-2}{t^2} + 3$

2. Compute the average value of $f(x) = \sqrt{2}x^{3/2} + 2e^{x/2}$ over the interval $[0, 2]$.

(a) $-\dfrac{11}{10} - \dfrac{1}{2}e$ (b) $-\dfrac{2\sqrt{2}}{5} + 4e - 4$ (c) $-\dfrac{2}{5} + 2e$ (d) $\dfrac{4}{5} + 4e$ (e) $\dfrac{8}{5} + 2e$

3. Estimate the integral $\int_0^1 \sqrt{x+2}\,dx$ by dividing the interval [0,1] into five equal segments and using the **Riemann sum** corresponding to **left endpoints**.

(a) $\frac{1}{5}(\sqrt{2.2}+\sqrt{2.4}+\sqrt{2.6}+\sqrt{2.8}+\sqrt{3.0})$
(b) $\frac{1}{5}(\sqrt{2.0}+\sqrt{2.2}+\sqrt{2.4}+\sqrt{2.6}+\sqrt{2.8})$
(c) $\frac{1}{10}(\sqrt{2.0}+\sqrt{2.25}+\sqrt{2.5}+\sqrt{2.75}+\sqrt{3.0})$
(d) $\frac{1}{5}(\sqrt{2.1}+\sqrt{2.3}+\sqrt{2.5}+\sqrt{2.7}+\sqrt{2.9})$
(e) none of the above

4. Find the **total area** (not the definite integral) between the function $f(x) = x^2 - 2$ and the x-axis over the interval $[0, 2]$.

(a) $\frac{8}{3}\sqrt{2} - \frac{4}{3}$
(b) $\frac{4}{3}$
(c) $\frac{-4}{3}\sqrt{2} + \frac{22}{3}$
(d) 2
(e) none of these

5. Compute the following indefinite integral $\int x^2(x^3-1)^{5/3}\,dx$.

(a) $\frac{3}{8}(x^3-1)^{8/3} + C$
(b) $\frac{1}{8}(x^3-1)^{8/3} + C$
(c) $\frac{1}{6}(x^3-1)^2 + C$
(d) $\frac{1}{8}x^3(x^3-1)^{8/3} + C$
(e) $\frac{2}{3}(x^3-1)^2 + C$

6. Compute the following definite integral $\displaystyle\int_4^6 \frac{2}{3x^2+3x-6}\,dx$.

(a) $\frac{2}{3}\ln\frac{5}{4}$ (b) $\frac{2}{3}\ln\frac{5}{8}$ (c) $\frac{2}{9}\ln\frac{15}{8}$ (d) $2\ln\frac{20}{9}$ (e) $\frac{2}{9}\ln\frac{5}{4}$

7. Compute the following indefinite integral: $\displaystyle\int 2t\ \ln t\ dt =$

(a) $\frac{t^2}{2}\ln t - \frac{t^2}{4} + C$ (b) $2\ln t - t^2 + C$ (c) $2\ln t + C$

(d) $t^2\ln t - \frac{t^2}{2} + C$ (e) $2 - 2\ln t + C$

8. The demand and supply curves for a product are given in the diagram. Use geometry to compute the **consumer surplus**.

(a) 120 (b) 240 (c) 60 (d) 360 (e) 180

9. The marginal cost at a given steel mill is $MC = 45t^{-1}$ in dollars per ton. If production is increased from 80 tons to 100 tons, what is the total change in cost?

(a) $45 \ln \frac{5}{4}$ (b) 162000 (c) $45 \ln 20$ (d) 900 (e) 4580

10. The demand function for designer shoe laces is $D(q) = -q^2 - 50q + 800$ and the supply function is $S(q) = q^2 + 10q$ (in thousands of units). Find the **equilibrium quantity** for designer shoelaces.

(a) $q_e = 2000$ (b) $q_e = 10$ (c) $q_e = 40$ (d) $q_e = 200$ (e) $q_e = 20$

11. The demand function for designer shoe laces is $D(q) = -q^2 - 50q + 800$ and the supply function is $S(q) = q^2 + 10q$ (in thousands of units). Compute the **producer surplus** for designer shoelaces.

(a) $\frac{2000}{3}$ (b) $\frac{15500}{3}$ (c) $\frac{9500}{3}$ (d) $\frac{3500}{3}$ (e) $\frac{500}{3}$

12. If Kate knows she can get 15% interest compounded continuously, how much money must Kate deposit now so that she can have \$1,000,000 in 40 years?

(a) $1000000e^6$ (b) $\dfrac{1000000}{.15}e^6$ (c) $1000000e^{-6}$ (d) $\dfrac{1000000}{.15}(e^6 - 1)$ (e) $\dfrac{1000000}{40}e^{-.15}$

13. The Jones family wants to buy a house to keep up with the neighbors. They decide to get a 30 year loan. If they think that they can find a 10% interest rate (compounded continuously) and make payments at a rate of \$5000 a year, how much can they afford to borrow now?

(a) $\$50000(1 - e^{-3})$ (b) $\$5000(e^3 - 1)$ (c) $\$5000(1 - e^{-3})$

(d) $\$50000(e^3 - 1)$ (e) $\$5000e^3$

Part II: Partial credit questions (8 points)

14. Now integrate $\int_1^2 2x\ e^{-2x} dx$. SHOW YOUR WORK

15. Consider the following integral: $\displaystyle\int_1^2 e^{2r}\,dr$.

(a) Estimate the integral using a **Riemann sum** with **midpoints** and $n = 4$.

(b) Now integrate to calculate the exact value of the integral.

(c) Use the answers from (a) and (b) to calculate how far off the approximation in (a) actually was from the correct answer.