Math 108 Exam 1 Solutions – Fall 2002

1. Integrating $\frac{dy}{dt} = \frac{4}{t^3}$ we obtain $y(t) = 4 \cdot \frac{-1}{2} \cdot t^{-2} + c$. Since $1 = y(1) = -2 \cdot \frac{1}{1^2} + c$, or $1 = -2 + c$, we have $c = 3$. Hence, we have $y(t) = -2 \cdot t^{-2} + 3$.

2. The average value of the given function $f(x)$ over $[0, 2]$ is

   $\frac{1}{2-0}\int_0^2 \sqrt{2}x^{3/2} + 2e^{x/2}dx = \frac{1}{2}\left(\frac{2\sqrt{2}}{5}x^{5/2} + 4e^{x/2}\right)_0^2 = \frac{1}{2}\left(\frac{16}{5} + 4e^1 - 0 - 4e^0\right) = -\frac{2}{5} + 2e$

3. $$\int_0^1 \sqrt{x+2}\ dx \approx \frac{(1-0)}{5}(\sqrt{0+2} + \sqrt{.2+2} + \sqrt{.4+2} + \sqrt{.6+2} + \sqrt{.8+2})$$
   $$= \tfrac{1}{5}(\sqrt{2.0} + \sqrt{2.2} + \sqrt{2.4} + \sqrt{2.6} + \sqrt{2.8})$$

4. The function $f(x) = x^2 - 2$ crosses the x-axis when $x^2 - 2 = 0$ or $x^2 = 2$ or $x = \pm\sqrt{2}$. $f(x) \leq 0$ on the interval $[0, \sqrt{2}]$ and $f(x) \geq 0$ on the interval $[\sqrt{2}, 2]$.

   Hence the total area $= \displaystyle\int_0^{\sqrt{2}} 0 - (x^2 - 2)dx + \int_{\sqrt{2}}^2 x^2 - 2dx = \left[\frac{-x^3}{3} + 2x\right]_0^{\sqrt{2}} + \left[\frac{x^3}{3} - 2x\right]_{\sqrt{2}}^2$

   $= \frac{-2\sqrt{2}}{3} + 2\sqrt{2} - 0 + \frac{8}{3} - 4 - (\frac{2\sqrt{2}}{3} - 2\sqrt{2}) = \frac{8}{3}\sqrt{2} - \frac{4}{3}$

5. $$\int x^2(x^3-1)^{5/3}\,dx = \int u^{5/3}\,\frac{du}{3} \qquad u = x^3 - 1 \qquad du = 3x^2dx$$
   $$= \tfrac{3}{8}u^{8/3} + C = \tfrac{3}{8}(x^3-1)^{8/3} + C$$

6. $$\int_4^6 \frac{2}{3x^2+3x-6}\,dx = \frac{2}{3}\int_4^6 \frac{1}{(x-1)(x+2)}\,dx$$
   $$= \frac{2}{3}\int_4^6 \frac{\frac{1}{3}}{x-1} + \frac{\frac{-1}{3}}{x+2}\,dx$$
   $$= \tfrac{2}{3}\left[\tfrac{1}{3}\ln|x-1| - \tfrac{1}{3}\ln|x+2|\right]_4^6$$
   $$= \tfrac{2}{9}\ln\left|\tfrac{x-1}{x+2}\right|_4^6$$
   $$= \tfrac{2}{9}\ln\tfrac{5}{8} - \ln\tfrac{1}{2} = \tfrac{2}{9}\ln\tfrac{5}{4}$$

   $$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$$
   $$1 = A(x+2) + B(x-1)$$
   $$1 = A(1+2) + B(1-1) = 3A$$
   $$1 = A(-2+2) + B(-2-1) = -3B$$

7. $$\int 2t\ \ln t\ dt = \int \ln t \cdot 2t\ dt \qquad u = \ln t \qquad dv = 2t\ dt$$
   $$= t^2\ln t - \int t^2\tfrac{1}{t}dt \qquad du = \tfrac{1}{t}dt \qquad v = t^2$$
   $$= t^2\ln t - \tfrac{t^2}{2} + C$$

8. Consumer Surplus=
$\frac{1}{2} \cdot (8-4) \cdot 30 = 60$

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9. Change in cost from 80 to 100 $= C(100) - C(80) = \int_{80}^{100} 45t^{-1}dt = 45 \ln t|_{80}^{100}$
$= 45(\ln 100 - \ln 80) = 45 \ln \frac{5}{4}$

10. $D(q) = S(q)$ means $-q^2 - 50q + 800 = q^2 + 10q$ so $2q^2 + 60q - 800 = 0$ or $2(q-10)(q+40) = 0$. Hence $q_e = 10$.

11. $q_e = 10$ so $p_e = S(10) = 10^2 + 10 \cdot 10 = 200$. Thus
$PS = \int_0^1 0 - q^2 - 50q + 800dq - 10 \cdot 200 = -\frac{q^3}{3} - 25q^2 + 800q|_0^{10} - 2000$
$= -\frac{1000}{3} - 2500 + 8000 - 0 - 2000 = \frac{9500}{3}$

12. The present value for a one-time investment with continuous interest is
$PV = FV\ e^{-rT} = 1000000e^{-.15 \cdot 40} = 1000000e^{-6}$

13. Solve for present value of a constant investment:
$PV = \frac{S}{r}(1 - e^{-rt}) = \frac{5000}{.1}(1 - e^{-.1 \cdot 30}) = 50000(1 - e^{-3})$

14. $\int_1^2 2x\ e^{-2x} dx = \int_1^2 2x \cdot e^{-2x} dx$ $\quad u = 2x \quad dv = e^{-2x}dx$
$= [2x\ \frac{-1}{2}e^{-2x}]_1^2 - \int_1^2 \frac{-1}{2}e^{-2x}2dx$ $\quad du = 2dx \quad v = \frac{-1}{2}e^{-2x}$
$= [-xe^{-2x}]_1^2 + \frac{-1}{2}e^{-2x}|_1^2$
$= -2e^{-4} + e^{-2} + \frac{-1}{2}e^{-4} + \frac{1}{2}e^{-2}$
$= -\frac{5}{2}e^{-4} + \frac{3}{2}e^{-2}$

15. (a) $\int_1^2 e^{2r}\, dr \approx \frac{(2-1)}{4}(e^{2 \cdot 1.125} + e^{2 \cdot 1.375} + e^{2 \cdot 1.625} + e^{2 \cdot 1.875})$
$= \frac{1}{4}(e^{2.25} + e^{2.75} + e^{3.25} + e^{3.75})$
$= 23.36044741$

(b) $\int_1^2 e^{2r}\, dr = \frac{1}{2}e^{2r}|_1^2 = \frac{1}{2}(e^4 - e^2)$
$= 23.60454697$

(c) Actual Error $= |\frac{1}{4}(e^{2.25} + e^{2.75} + e^{3.25} + e^{3.75}) - \frac{1}{2}(e^4 - e^2)|$

$$= |23.36044741 - 23.60454697|$$

$$= .2440995600$$