Mathematics 108, Calculus II for Business

Summer Session 1997

Exam 1

Friday, July 11, 1997

This Exam is worth a total of 100 points. Point values are assigned next to the problem numbers. You should have a total of 7 sheets of paper - one cover page, five exam pages and one blank sheet for scratch work. All the problems are partial credit problems. Please show your work in the test booklet; you do not have to turn in your scratch paper. Calculators, books and notes are not allowed. The exam will begin at 8:35 and end at 10:35.

Please sign the pledge: "On my honor, I have neither given nor received unauthorized aid on the test."

GOOD LUCK!

1. (8 points) Determine the average value of the function $h(t) = 4x^3 - 4x + 5$ between $x = -1$ and $x = 2$.

2. (8 points) Find the area under the curve $y = 4e^{-3x}$ for $x \geq 1$.

3. (20 points)

a) Suppose that a company determines that its marginal cost for producing x items is given by the function $MC(x) = 3x^2 - 6x + 1$. If the fixed cost is 20 monetary units, determine the total cost function C(x) for the company.

b) Suppose that the company can sell each item for 10 monetary units. Assuming that each item produced is sold, what is the total revenue function R(x) for the company when it produces x items? Remember that revenue is given by price times quantity.

c) Knowing the total cost and the total revenue for producing x items, write down an equation for the total profit for producing x items.

d) What production level will maximize profit for the company?

4. Evaluate the following integrals: (6 points each)

a) $\int (4x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{-3}{2}})\, dx$

b) $\int (x-1)e^{3x^2-6x}\, dx$

c) $\int (x^3 lnx)\, dx$

d) $\int \sin x \cos x\, dx$
(Note: There are two different ways to do this problem. If you get stuck try a different integration method.)

5. (6 points) Suppose that someone wishes to invest $20,000 in a bank at 10% interest compounded semi-annually (twice a year). What would be the future value of this investment after 20 years? Simplify your answer as much as possible but do **not** try to solve for a specific number.

What would be the future value assuming that the interest was compounded continuously? You may assume that $e^2 \approx 7.4$.

6. (4 points) At the end of the five years you would like to purchase a car for $17,000. What initial amount (present value) would you need to invest if the interest rate is 11% compounded annually? Set up the equation but do **not** evaluate it.

7. (10 points) Find the area between the functions $y = x^2 + 3$ and $y = -5x - 3$. You do not need to sketch the graphs of these functions, but remember that area must be a positive number.

8. (20 points)

Suppose that the demand curve for producing q widgets is given by:

$$D(q) = \frac{50}{q+1}$$

and the supply curve in given by:

$$S(q) = q + 6$$

a) What is the equilibrium quantity, q_e? (Remember that the equilibrium point is where the supply curve meets the demand curve.)

b) What is the equilibrium price, p_e?

c) What is the consumer surplus?

d) What is the producer surplus?