Name: **Teacher:** Karen S. Brown

Mathematics 108, Calculus II for Business

Summer Session 1997

Exam 2

Tuesday, July 22, 1997

This Exam is worth a total of 100 points. Point values are assigned next to the problem numbers. You should have a total of 10 sheets of paper - one cover page, one page of formulas, seven exam pages and one blank sheet for scratch work. All the problems are partial credit problems. Please show your work in the test booklet; you do not have to turn in your scratch paper. Calculators, books and notes are not allowed. The exam will begin at 8:35 and end at 10:35.

Please sign the pledge: "On my honor, I have neither given nor received unauthorized aid on the test."

GOOD LUCK!

Useful formulas:

A. The equation for the plane tangent to the function f(x,y) at the point (x_0, y_0):

$$z = f(x_0, y_0) + \frac{\partial f}{\partial x}(x_0, y_0)(x - x_0) + \frac{\partial f}{\partial y}(x_0, y_0)(y - y_0).$$

B. The equation for the discriminant $D(x_0, y_0)$ used in the Second Derivative Test for functions of two variables:

$$D(x_0, y_0) = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial y \partial x}\right)^2$$

C. For the 2x2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the inverse matrix A^{-1} is given by $A^{-1} = \frac{1}{D}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, where D is the determinant of the matrix A.

1. Solve the following differential equations:(3 points each)

a. $y' = e^{2t} - 1$

b. $y' = \frac{-3}{2}y, \qquad y(0) = 17$

2. Solve the following systems of equations, and describe their solutions geometrically. (4 points each)

a)

$$\begin{aligned} 2x - 6y &= 5 \\ -4x + 7y &= -3 \end{aligned}$$

b)

$$\begin{aligned} -3x + 5y &= 13 \\ 9x - 15y &= 42 \end{aligned}$$

3. Find the partial derivatives, first with respect to x and then with respect to y of the function $f(x, y) = e^{x^2 y}$. Then find the equation of the plane tangent to the surface at the point $(2, 3, e^{12})$. (8 points)

4. Given the following matrices A and B below, compute the products AB and BA. If the product is undefined, state this fact. (5 points)

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 4 & -2 \\ -1 & -3 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & -1 & 1 \\ 0 & 4 & -3 \end{pmatrix}$$

5. A certain individual has a continuous income stream of $20,000 per year which is invested at 10% interest compounded continuously. What will be the future value of this investment after 10 years? You may assume that $e \approx 2.71$. (5 points)

6. Solve the following separable differential equation. (6 points)

$$y' = y^2 - t^2 y^2$$

7. Find the critical points of the function $f(x, y) = x^2 + y^3 - 6y^2 + 6x - 15y$. (7 points)

8. Suppose that an individual wishes to invest an initial amount of money into a bank account with an interest rate of 10% compounded continuously. The person needs three times that amount at the end of 8 years. Will the investment triple by this time? You may assume that $\ln 3 \approx 1.1$. (5 points)

9. Write the following system of linear equations in matrix form; i.e. in the form $Ax = b$, where A is a 2x2 matrix, and x and b are 2x1 matrices. Then solve the system of equations by finding A^{-1} and multiplying on both sides of the equation. (6 points)

$$-4x + 7y = 6$$
$$-2x + 3y = -4$$

10. A company produces two items, call them Item A and Item B. The company would like to determine whether to increase the production of one item or the other, but not both. The profit function for the company is

$$P(x, y) = 6x^2 - 3x + y^2 - 20y + 7,$$

where x is the amount of Item A produced and y is the amount of Item B produced. The company is currently producing 4 units of Item A and 8 units of Item B. Using this information, compute the partial marginal profit for increasing production first for Item A (keeping Item B's production constant) and then for Item B (keeping Item A's production constant.) What would be your recommendation to the company? Should it increase production for either item? (8 points)

11. Determine whether or not the following 3 x 3 matrices have an inverse by evaluating the determinant. State your conclusions. (6 points)

$$A = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 3 & -2 \\ 0 & 4 & -3 \end{pmatrix} \qquad B = \begin{pmatrix} -3 & 1 & 2 \\ 0 & 6 & 0 \\ 6 & -2 & -4 \end{pmatrix}$$

12. Solve the following system of linear equations using Gaussian elimination. (10 points)

$$\begin{aligned} x - 3y + 4z &= 7 \\ -3x + 5y - 2z &= -3 \\ 4x - 4y + z &= 7 \\ x + y - z &= 4 \end{aligned}$$

13. Solve the following separable differential equation with the given initial value: (10 points)

$$y' = \frac{te^{t^2}}{y}, \qquad y(0) = 1$$

14. It is given that (0,0) is a critical point of the function $Q(x, y) = x^2y + 3xy^2 - 6y^2 - 3x^2$. Determine whether there is a relative maximum, relative minimum or a saddle point at (x,y)=(0,0), if possible. (10 points)