Name: **Teacher:** Karen S. Brown

Mathematics 108, Calculus II for Business

Summer Session 1997

Final Exam

Friday, July 30, 1997

This Exam is worth a total of 150 points. Point values are assigned next to the problem numbers. You should have a total of 11 sheets of paper - one cover page, nine exam pages and one blank sheet for scratch work. All the problems are partial credit problems. Please show your work in the test booklet; you do not have to turn in your scratch paper. Calculators, books and notes are not allowed. The exam will begin at 8:35 and end at 10:35.

Please sign the pledge: "On my honor, I have neither given nor received unauthorized aid on the test."

GOOD LUCK!

ENJOY THE REST OF THE SUMMER!

1. Suppose that a company determines that its marginal profit function is given by $MP(x) = 6x^2 - 2x + 3$, where x is the number of items produced. If the company makes a profit of \$1000 by producing 10 items, determine the total profit function. (5 points)

2. Let $A = \begin{pmatrix} -1 & 2 & 0 \\ 3 & 1 & -2 \\ 2 & -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 0 & 5 \\ 1 & -2 \end{pmatrix}$.

Compute AB. (5 points)

Compute the determinant of A, det(A). Does the matrix A have an inverse? (5 points)

3. A three-year-old business is analyzing its profit growth from the past three years. You have been asked to help them with this assessment. After 1 year the business made one profit unit; after two years it made 3 profit units and after 3 years it made 7 profit units. Use the method of least squares to fit a linear function to these data points. (Hint: Plot the data in a graph so that you know you have the correct data points.) (12 points)

If the business asks you to predict what its profit would be at the end of the fourth year, what would you tell them? (3 points)

4. Joe has a lawn mowing job. If he completes the work he earns \$40. But there is a 30% chance it may rain, in which case he won't finish the job. He can pay Jane \$20 to help him and ensure that he finishes the job. If X is the amount Joe will get if he does not get Jane to help, calculate E(X) and thus decide whether Joe should hire Jane or not. (If it rains, assume Joe will make no money and if Joe hires Jane assume they will be able to finish the job before it rains.) (6 points)

5. Given the following table of outcomes and probabilities, compute the expected value. Then set up the equation for the variance, but **do not evaluate it** . (6 points)

Outcome:	1	2	3	4
Probability:	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

6. Solve the following separable differential equation with the given initial value. (7 points)

$$\frac{dy}{dt} = y(t^2 + 4t), \qquad y(0) = 1$$

7. Solve the following system of equations using Gaussian elimination. (10 points)

$$\begin{aligned} x - 3y + 2z &= 6 \\ -3x + 2y - z &= 0 \\ -2x - y + 3z &= 8 \end{aligned}$$

8. Evaluate the following integral. (6 points)

$$\int x \cos 5x \, dx$$

9. The riders of the New Town Elementary school bus consists of 5 five year olds, 3 six year olds, 10 eight year olds, 1 nine year old, 4 eleven year olds and a twelve year old. A child is selected at random and her age is noted. Let X be the outcome. Construct a probability table of X. (6 points)

What is the probability that the selected child is under 7 years of age? (2 points)

What is the probability that the selected child is over ten? (2 points)

10. Suppose that you have \$10,000 you would like to invest in a bank at a rate of 10% compounded continuously. How much will this investment be worth at the end of 5 years? You may assume that $\sqrt{e} \approx 1.65$. (5 points)

11. A business produces two products A and B. Let x and y denote, respectively, the quantity of A and B to be produced. Limitations on the company resources require that $500x^2 + 100y^2$ be at most 840,000. Each unit of A yields a $5000 profit and each unit of B yields a $500 profit. What should x and y be to yield a maximum profit? (Note: You are maximizing the **profit** in this problem under a certain constraint.) (20 points)

12. Suppose that the demand function for a certain item is given by $D(q) = \frac{20}{q+5}$ and the supply curve is given by $S(q) = q - 3$.

a. What is the equilibrium price and the equilibrium quantity? (5 points)

b. Set up the integral for the consumer surplus for the given demand function, but **do not evaluate it.** (3 points)

13. A random variable X is exponentially distributed with mean 2.
a). What is the probability density function of this random variable? (3 points)

b).Determine $P(1 \leq X \leq 3)$. (3 points)

14. Your company produces two products, call them Item 1 and Item 2. Suppose that you have determined that your costs are given by $C(x, y) = x^2 + 100x + y^2 - 300y + 3000$, where x is the number of units of Item 1 produced and y is the number of units of Item 2 produced. If Item 1 sells for \$500 and Item 2 sells for \$300, what production levels will maximize the profit? Hint: Find the revenue function, then determine the profit function from knowing the revenue and the cost.

Recall that $D(x_0, y_0) = \left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2$.

(20 points)

15. Evaluate the following integral: (6 points)

$$\int (3x^2 + 5)e^{x^3+5x}\, dx$$

16. Suppose $f(x) = k(x^2 + 2x)$ is a probability density function for a continuous random variable on the interval $0 \leq x \leq 3$.

a. Find the value for k. (5 points)

b. Find the corresponding cumulative distribution function. (5 points)