[12pt]letter amsmath 6.5in0in 9.5in-1.0in

document **MULTIPLE CHOICE SECTION - EACH PROBLEM IS WORTH 5 POINTS**

(In Problems 1-10 mark the correct answer on the front cover)

1. A cube having volume of 64 units sits in 3-space with each edge parallel to an axis. The center of one of the sides of the cube is at the point $(1, 1, 1)$. Which of the following points could be a corner of the cube?

(a) $(-1, 1, 1)$ (b) $(-1, 3, 3)$ (c) $(-3, 3, 5)$ (d) $(3, 1, -1)$ (e) $(5, 1, -3)$

2. Suppose that $f(x, y)$ is a function with $\partial f \partial x = 3x^2 + e^{xy} - y$. Find $\partial^2 f \partial x^2$.

(a) $6 + y^2 e^{xy}$ (b) $xe^{xy} - 1$ (c) $6x + xe^{xy} - 1$ (d) $6x + e^{xy}$ (e) $6x + ye^{xy}$

3. Suppose that $f(x, y)$ and $g(x, y)$ are both linear functions. Which of the following must also be linear functions:

I. $f(3x, y)$ II. $f(x, y)g(x, y)$ III. $2f(x, y) - g(x, y)$ IV. $xf(x, y)$ V. $g(x, xy)$

(a) I and III (b) only IV (c) II and VI (d) only III (e) III and V

4. Widgets-R-Us makes two kinds of widgets: type X and type Y. The profit from selling x widgets of type X and y widgets of type Y is a function $P(x, y)$. Suppose that

$$P(100, 150) = \$10,000, \quad \partial P \partial x(100, 150) = 200, \quad \textit{and} \quad \partial P \partial y(100, 150) = -100.$$

Estimate the profit the company will earn if it sells 120 widgets of type X and 160 widgets of type Y.

(a) \$11,000 (b) \$13,000 (c) \$12,000 (d) \$15,000 (e) \$14,000

5. For what value(s) of k will the following system of equations have infinitely many solutions:

$$x + y + z = 3$$

(a) $k = -2$ (b) $k = 2, 4$ (c) no values of k (d) all $k \neq -2$ (e) $k = 1$

6. Which of the following is the function which must be minimized in order to find the Least Squares Fit straight line ($y = ax + b$) through the three points $(0, 3)$, $(1, 1)$, $(4, 2)$?

(a) $E = (3a + b)^2 + (a + b - 1)^2 + (2a + b - 4)^2$

7. In 3-dimensions a line can be described by three equations such as the following:

$$x = 1 + t,\ y = 2,\ z = 3,$$

where t is allowed to be any real number. i.e. for each value of t, we get a point $(1 + t, 2, 3)$ on the line. Find the value(s) of t for which the corresponding point on the line is at a distance of $\sqrt{5}$ units from the point $(1, 2, 2)$.

(a) $t = -3, 1$ (b) $t = -1$ (c) $t = -2, 2$ (d) $t = -\sqrt{24}, \sqrt{24}$ (e) $t = 3$

8. Find the equation of the tangent plane to the graph of the function $f(x, y) = -2xy$ at the point $(1, -1)$.

(a) $z = 2x - 2y + 2$ (b) $z = -2x + 2y + 2$ (c) $z = -2x - 2y + 2$
(d) $z = 2x - 2y - 2$ (e) $z = 2x + 2y + 2$

9. A function $f(x, y)$ satisfies $\partial f \partial x = 0 = \partial f \partial y$ at the point $(7, 10)$. Suppose also that $\partial^2 f \partial x^2 = -3$, $\partial^2 f \partial y^2 = 4$, and $\partial^2 f \partial x \partial y = -4$ at the point $(7, 10)$. Which of the following is true?

(a) $(7, 10)$ is a relative maximum point.

10. Write the equation of the plane which contains the points $(-2, 0, 0)$, $(0, 3, 0)$, and $(0, 0, 6)$.

(a) $z = 3x - 2y + 6$ (b) $z = -13x + 12y + 3$ (c) $z = 12x - 13y - 2$

(d) $z = -3x + 2y + 6$ (e) $z = -2x + 3y + 6$

PARTIAL CREDIT PART

(In Problems 11-15 show all work on the paper)

11. (8 points) Consider the function $f(x, y) = 3x^2 + 3y^2$. Sketch the level curves for $f(x, y)$ at the values $z = -3, 0, 3, 9$ on the following graph. Label each curve. If a level curve doesn't exist, say so and explain why.

12. (7 points) The profit from selling x units of one item and y units of another item is given by $P(x, y) = 3x^4 - x^2y + xy^2 - y$. Assuming the company will produce a total of 100 items, use the Method of Lagrange multipliers to set up the system of equations whose solution gives the possible points at which the profit could be maximized. You do **not** have to solve the system.

13. (10 points) Find the linear function $f(x, y)$ which satisfies $f(1, 1) = 5$, $f(0, 1) = 3$, and $f(1, 2) = 6$.

14. (10 points) Identify all pairs (x, y) which satisfy the following system of equations:

$$2y = 4x\lambda$$

15. (15 points) From the Method of Least squares, the regression line $y = ax + b$ which best fits the three points $(0, 2)$, $(1, -1)$, and $(3, 0)$ can be determined by finding the point (a, b) where the function

$$E(a, b) = 5a^2 + 6ab + 3b^2 + 2a - 2b + 5$$

is a minimum.

(a) Find all the critical point(s) of $E(a, b)$.

(b) Apply the second derivative test to each critical point to determine whether it is a relative maximum, relative minimum, or saddle point.