Name: ______________________

Math 108 - Calculus II for Business

Final Exam - Fall Semester 1998

Wednesday, December 16, 1:45-3:45 p.m.

This Examination contains **34** problems, worth a total of 200 points. Books and notes are not allowed. You may use your calculator.

The first **three** problems are partial credit problems worth a total of 45 points. For these problems, **show** your computations and **clearly** mark your answers on the page. The remaining problems are multiple choice with no partial credit, and each is worth 5 points. Record your answers to these problems by placing an $\times$ through one letter for each problem below:

4.	a	b	c	d	e	**15.**	a	b	c	d	e	**25.**	a	b	c	d	e
5.	a	b	c	d	e	**16.**	a	b	c	d	e	**26.**	a	b	c	d	e
6.	a	b	c	d	e	**17.**	a	b	c	d	e	**27.**	a	b	c	d	e
7.	a	b	c	d	e	**18.**	a	b	c	d	e	**28.**	a	b	c	d	e
8.	a	b	c	d	e	**19.**	a	b	c	d	e	**29.**	a	b	c	d	e
9.	a	b	c	d	e	**20.**	a	b	c	d	e	**30.**	a	b	c	d	e
10.	a	b	c	d	e	**21.**	a	b	c	d	e	**31.**	a	b	c	d	e
11.	a	b	c	d	e	**22.**	a	b	c	d	e	**32.**	a	b	c	d	e
12.	a	b	c	d	e	**23.**	a	b	c	d	e	**33.**	a	b	c	d	e
13.	a	b	c	d	e	**24.**	a	b	c	d	e	**34.**	a	b	c	d	e
14.	a	b	c	d	e												

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam".

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PARTIAL CREDIT PART

In problems 1-3, show all work on the paper. You **MUST** show your work to receive full credit for these problems.

1. (15 points) Consider the functions $f(x) = 2x$ and $g(x) = x^2 - x$.

(a) Find the x-values of the points where the graphs of $f(x)$ and $g(x)$ intersect.

(b) Sketch the graphs of $f(x)$ and $g(x)$ using the same coordinate system. Shade the region enclosed by the graphs of $f(x)$ and $g(x)$.

(c) Write in simplified form the definite integral that gives the area of the region enclosed by the graphs of $f(x)$ and $g(x)$. You do *not* need to compute the integral.

2. (15 points) For a certain item, the demand curve is $D(q) = -2q + 12$, and the supply curve is $S(q) = 3q + 2$.

(a) Find the equilibrium price and the equilibrium quantity.

(b) Find the consumer surplus.

(c) Find the producer surplus.

3. (15 points) In the weeks before Christmas, the number of toys made each day at Santa's workshop is normally distributed with a mean of 500 and standard deviation of 50. Compute the probability that on a random day between 475 and 550 toys are made.

MULTIPLE CHOICE SECTION - EACH PROBLEM IS WORTH 5 POINTS

In problems 4-34, mark the correct answer on the front cover.

4. An investment is growing at the *rate* of $R(t) = 20t + 5$ dollars per year. If \$1000 was initially invested, find the value of the investment after 5 years.

(a) \$1325 (b) \$1300 (c) \$1025 (d) \$1275 (e) \$1225

5. Solve the initial value problem

$$\frac{dy}{dx} = 3x^2 + 1, \quad y(1) = 2.$$

(a) $y = x^3 + x + 2$ (b) $y = 3x^3 + x + 2$ (c) $y = x^3 + x$

(d) $y = 3x^3 + x$ (e) $y = x^3 + x - 2.$

6. Which of the following methods would you use to compute

$$\int xe^{x^2}\,dx.$$

(a) Integration by parts with $u = e^{x^2}$ and $v' = x$.
(b) Integration by parts with $u = x$ and $v' = e^{x^2}$.
(c) Partial fractions.
(d) Substitution with $u = x^2$.
(e) Direct integration using basic formulas.

7. Write the equation of the plane that contains the points $(0,0,3)$, $(2,0,1)$, $(0,2,1)$.

(a) $z = x + y + 3$ (b) $z = -x - y + 3$ (c) $z = x + y - 3$

(d) $z = -3x - 3y - 3$ (e) None of the preceding

8. For which value(s) of k will the following system of equations have no solution:

$$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y + 2z &= 5 \\ 2x + 4y + kz &= 5 \end{aligned}$$

(a) $k = 0, -1$ (b) $k = 1$ (c) $k = -1$ (d) $k = 2$ (e) None of the preceding

9. Find the equation of the tangent plane to the graph of the function $f(x,y) = x^2 - y$ at (1,0).

(a) $z = 2x - y$ (b) $z = 2x - y + 1$ (c) $z = 2x - y - 1$

(d) $z = x - 2y + 1$ (e) $z = x - 2y$.

10. Compute $\int x \ln x dx$ using integration by parts.

(a) $x^2 \ln x - \frac{x^2}{2} + C$ (b) $\frac{x^2}{2} \ln x + \frac{x^2}{4} + C$ (c) $\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$

(d) $\frac{x^2}{2} - \frac{x^2}{4} \ln x + C$ (e) None of the preceding

11. Suppose $f(x)$ is a function which takes the following values:

x	0	1	2	3	4
f(x)	.1	.4	.5	1	-.1

Estimate $\int_0^4 f(x)dx$ by computing the Riemann sum with 4 subintervals and left endpoints.

(a) 1 (b) 1.9 (c) 1.45 (d) 1.1 (e) 2

12. Find the present value of a perpetual income stream flowing continuously at a rate of $\$20,000$ per year, which is earning continuously compounded interest at 8%.

(a) \$140,000 (b) \$160,000 (c) \$250,000 (d) \$200,000 (e) \$14,000

13. Find the partial fraction decomposition of

$$\frac{1}{x^2 - 6x + 5}.$$

That is find the numbers A and B such that:

$$\frac{1}{x^2 - 6x + 5} = \frac{A}{x-5} + \frac{B}{x-1}.$$

(a) $A = 1, B = -1$
(b) $A = -\frac{1}{4}, B = \frac{1}{4}$
(c) $A = \frac{1}{4}, B = -\frac{1}{4}$
(d) $A = -\frac{1}{3}, B = \frac{1}{3}$
(e) $A = \frac{1}{3}, B = -\frac{1}{3}$

14. Find the solution of the following initial value problem:

$$\frac{dy}{dt} = 4y^2t^3, \quad y(0) = -1.$$

(a) $y = -\dfrac{1}{4t^4+1}$
(b) $y = -\dfrac{1}{t^4+1}$
(c) $y = \dfrac{1}{4t^4+1}$
(d) $y = \dfrac{1}{t^4+1}$
(e) None of the preceding

15. Compute the improper integral:

$$\int_1^\infty \frac{2x}{(x^2+1)^2}dx.$$

(a) 1/2 (b) 1/4 (c) 1 (d) 0 (e) None of the preceding

16. Solve the following equation for p in terms of t.

$$\frac{1}{2}\ln\left(\frac{p-1}{p}\right) = 2t+1.$$

(a) $p = \dfrac{1}{1-e^{4t+2}}$ (b) $p = \dfrac{1}{1+e^{4t}}$ (c) $p = \dfrac{1}{1-e^{4t}}$ (d) $p = \dfrac{1}{1+e^{4t+2}}$ (e) $p = e^{-2-4t}$

17. The level curve for the function $f(x,y) = (2x^2+2y^2)^2$ with height $z = 1$ is which of the following?

(a) a line (b) a circle (c) a point (d) a parabola (e) does not exist

18. An individual opens an account with initial amout of $\$100,000$, and then makes continuous withdrawals at the rate of $\$2,000$ per year. We assume an interest rate of 5% compounded continuously. Find the differential equation and initial condition that models the amount of money $M(t)$ in the account.

(a) $\frac{dM}{dt} = .05M - 2000,\ M(0) = 100,000$ (b) $\frac{dM}{dt} = -.05M - 2000,\ M(0) = 100,000$

(c) $\frac{dM}{dt} = .05M + 2000,\ M(0) = 98,000$ (d) $\frac{dM}{dt} = -.05M + 2000,\ M(0) = 98,000$

(e) $\frac{dM}{dt} = 2000M - 50000,\ M(0) = 100,000$

19. According to the method of Lagrange multipliers, at which of the following points (x, y) could the function $f(x, y) = 2x^2 - y^2$ possibly obtain a maximum value subject to the constraint $x + y = 2$.

(a) $(-2, 4)$ (b) $(2, -4)$ (c) $(0, -2)$ (d) $(1, -2)$ (e) $(-1, 2)$

20. A function $f(x, y)$ satisfies $\dfrac{\partial f}{\partial x} = 0 = \dfrac{\partial f}{\partial y}$ at the point $(0, 1)$. Suppose also that $\dfrac{\partial^2 f}{\partial x^2} = 1$, $\dfrac{\partial^2 f}{\partial y^2} = 4$, $\dfrac{\partial^2 f}{\partial x \partial y} = 3$ at the point $(0, 1)$. Which of the following is true?

(a) (0,1) is a relative maximum point
(b) (0,1) is a saddle point
(c) (0,1) is a relative minimum point
(d) (0,1) is not a critical point
(e) (0,1) is a critical point, but the second derivative test is inconclusive

21. Suppose that $f(x, y)$ is a function which satisfies $f(1, 2) = 4$, $\dfrac{\partial f}{\partial x}(1, 2) = 2$, and $\dfrac{\partial f}{\partial y}(1, 2) = 3$. Use linear approximation to estimate $f(.8, 2.1)$.

(a) 4.1 (b) 3.9 (c) 4.2 (d) 3.8 (e) 4.3

22. What bowl game are the Irish going to? *Hint:* It begins with a G.

(a) Orange (b) Alamo (c) GATOR (d) Sugar (e) Rose

23. Suppose that an experiment has sample space $S = \{s_1, s_2, s_3\}$ and $P(s_1) = .7$. Which of the following *could* be $P(s_2)$?

(a) $-.5$ (b) .1 (c) .4 (d) .8 (e) 1.2

24. Santa has 9 reindeer. One has a red nose, three have brown noses, and five have black noses. If Santa picks one reindeer at random, what is the probability that the reindeer's nose is brown?

(a) 1/9 (b) 1/5 (c) 3/5 (d) 1/3 (e) 5/9

25. Suppose that the sample space for an experiment is $S = \{s_1, s_2, s_3, s_4\}$ and that $P(s_1) = .2$, $P(s_2) = .1$, and $P(s_2, s_3) = .5$. Find $P(s_4)$.

(a) .2 (b) 0 (c) .7 (d) .4 (e) .3

26. Let X be a random variable with values 1, 2, 5, and 7, with probabilities given by the following table:

X	1	2	5	7
P	.3	.2	.4	.1

What is the expected value of X?

(a) 3.75 (b) 3.5 (c) 3.4 (d) 2.8 (e) 3

27. Consider the experiment of choosing an integer i at random from $\{1, 2, 3, 4\}$. Let X be the random variable given by $X(i) = (i - 3)^2$ for each integer i. Find $P(X = 1)$.

(a) 3 (b) 1/4 (c) 0 (d) 3/4 (e) 1/2

28. Let X be a continuous random variable with density function $f(x) = \dfrac{1}{2\sqrt{x}}$ on $1 \leq x \leq 4$. Which of the following expressions is $P(X \leq 3)$?

(a) $\displaystyle\int_1^3 \frac{1}{2\sqrt{x}}dx$ (b) $\displaystyle\int_3^\infty \frac{1}{2\sqrt{x}}dx$ (c) $\displaystyle\int_1^3 \sqrt{x}dx$

(d) $\displaystyle\int_3^4 \sqrt{x}dx$ (e) $\displaystyle\int_3^4 \frac{1}{2\sqrt{x}}dx$

29. Let X be a continous random variable with density function $f(x) = \frac{c}{4x^2}$ on the interval $1 \le x \le 5$. Find the constant c.

(a) $c = -4$ (b) $c = 1$ (c) $c = -5$ (d) $c = 5$ (e) $c = 4$

30. Let X be a continuous random variable with density function $f(x) = \frac{1}{2\sqrt{x}}$ on $1 \le x \le 4$, which of the following expressions is $E(X)$?

(a) $\int_1^4 \frac{1}{2}\sqrt{x}dx$ (b) $\int_1^4 \frac{1}{2\sqrt{x}}dx$ (c) $\int_0^\infty \frac{1}{2}\sqrt{x}dx$

(d) $\int_0^\infty \frac{1}{2\sqrt{x}}dx$ (e) $\int_1^4 \frac{1}{2}x^{\frac{3}{2}}dx$

31. Let X be a *continuous* random variable on $1 \le x < \infty$, what is $P(X = 3)$?

(a) -1 (b) 1 (c) 0 (d) 1/3 (e) 1/2

32. Suppose that X is a continuous random variable on $1 \leq x \leq 5$ and that the *cumulative distribution* function is $F(x)$. Suppose further that $F(3) = 1/3$. Find $P(3 \leq X \leq 5)$.

(a) 1/3 (b) 3/5 (c) 2/5 (d) 2/3 (e) 1

33. Let Z be the standard normal random variable with $\mu = 0$ and $\sigma = 1$.
Find $P(0 \leq Z \leq 2.25)$.

(a) .4878 (b) .4861 (c) .9878 (d) .9938 (e) .0122

34. Let X be a normal random variable with a mean of 6 and standard deviation of 4. In terms of probabilities of the standard normal random variable Z, which of the following is $P(-2 \leq X \leq 10)$?

(a) $P(-2 \leq Z \leq 10)$ (b) $P(-1 \leq Z \leq 1)$ (c) $P(4 \leq Z \leq 6)$

(d) $P(-6 \leq Z \leq 6)$ (e) $P(-2 \leq Z \leq 1)$