Mathematics 108

Exercises 6.2

In Exercises 1 through 4 estimate the first-order partial derivatives from the given values of each function $f(x,y)$, at the given point.

1. $f(0,0)=0,\ f(0.1,0)=0.3,\ f(0,0.2)=-0.4;\ (0,0)$

2. $f(1,2)=8,\ f(1.5,2)=10,\ f(1,2.6)=8.3;\ (1,2)$

3. $f(-4,5)=-7,\ f(-3.8,5)=-6,\ f(-4,5.4)=-7.2;\ (-4,5)$

4. $f(4.5,6)=20,\ f(5,5.7)=23,\ f(5,6)=21;\ (5,6)$

In Exercises 5 through 8 compute the indicated partial derivatives of each function at the given points.

5. $\dfrac{\partial f}{\partial x}(2,1)$ if $f(x,y)=x^2+xy-3y^2+4$

6. $\dfrac{\partial f}{\partial y}(1,0)$ if $f(x,y)=x^3e^{-y^2}-x$

7. $\dfrac{\partial f}{\partial x}(25,16)$ if $f(x,y)=100x^{\frac{1}{2}}y^{\frac{1}{2}}$

8. $\dfrac{\partial f}{\partial y}(0,0)$ if $f(x,y)=\ln\,(x^2+y^2+1)$.

In each Exercise 9 through 22 find the first order partial derivatives of the fiven function. Assume that the variables are restricted on a domain on which the function is well defined.

9. $f(x,y)=5-8x+5y$

10. $f(x,y)=3x^2+xy-2$

11. $f(x,y)=x^3y^2-2x^2y+y$

12. $f(x,y)=y^3e^{2xy}-x^5$

13. $M(r,t)=1000e^{rt}$

14. $f(x,y)=\sqrt{x^2+y^2}$

15. $R(s,t)=\frac{5t^2}{s}$

16. $d(a,b)=(3a+b-5)^2$

17. $l(x,y)=\ln\,(x^2-5y+3)$

18. $f(x,y)=\ln\,(x^2+1)e^{-xy^2}$

19. $Q(K,L)=10K^{0.4}L^{0.6}$

20. $f(x,y)=16x^{\frac{1}{8}}y^{\frac{7}{8}}$

21. $M(t,r) = 1000\ (1 + \frac{r}{360})^{360t}$

22. $P(t,K) = \frac{K}{1+e^{-0.02t}}$

For each function in Exercises 23 through 28 find the equation of the tanger + plane to its graph at the given point, as well as it's linear aproximation at the same point.

23. $f(x,y) = 3x^2 + 5y^2;\ (1,2)$

24. $f(x,y) = x^2 - 3y^2;\ (-1,2)$

25. $f(x,y) = e^{x^2-y^4};\ (1,-1)$

26. $f(x,y) = \ln\ (x^2 + 3y^2 + 2);\ (0,0)$

27. $f(x,y) = 8x^{\frac{1}{2}}y^2;\ (4,1)$

28. $f(x,y) = x^2e^{yx} + y^3;\ (0,-2)$

Exercises 6.3

In Exercises 1 through 6 find the critical points of the given function.

1. $f(x,y)=x^2+y^2-2x+4y+6$

2. $f(x,y)=2y^2+x^2+xy+3x-2y+1$

3. $f(x,y)=y^2-5xy-2x^2+3$

4. $f(x,y)=x^2=3y^2=3y-2x+8$

5. $f(x,y)=3xy-x^3-y^3+10$

6. $f(x,y)=y^3+3x^2-6xy-9y-2$

In Exercises 7 through 18 use the first and second derivative tests to find the local minima, maxima, and saddle points of the given function. If the second derivative test is inconclusive, so state.

7. $g(x,y)=x^2-xy+y^2+3y-1$

8. $f(x,y)=2y^2-xy+x^2-7x+4$

9. $f(x,y)=x^2-xy+2y+x+3$

10. $f(x,y)=8+2y-x^2-y^2$

11. $f(x,y)=x^3+y^3-3xy+4$

12. $f(x,y)=y^3-x^3-3xy+5$

13. $f(x,y)=8+4x+2y-x^2-y^2$

14. $f(x,y)=xy-\frac{1}{2}x^2-\frac{1}{3}y^3$

15. $f(x,y)=x^4+y^4$

16. $f(x,y)=x^4-y^4$

17. $f(x,y)=1-x^4-y^4$

18. $f(x,y)=1+x^4+y^4$

In exercises 19 through 22 the given $P(x,y)$ is the profit function of a company selling x units of a product X and y units of another product Y. In each case find the values of x and y that maximize the company's profit, as well as the maximum profit.

19. $P(x,y)=8x+11y-900-0.01(x^2+xy+2y^2)$

20. $P(x,y)=12x+9y-500-0.01(4x^2+xy+y^2)$

21. $P(x,y)y-10,000-0.01(2x^2+xy+3y^2)$

22. $P(x,y)=150x+320y-15,000-0.01(3x^2+xy+5y^2)$

Answers of Exercises 6.3

1. $(1,-2)$ **2**. $(-2,1)$ **3**. $(0,0)$ **4**. $(1,-\frac{1}{2})$ **5**. $(0,0),\ (1,1)$

6. $(-1,-1),\ (3,3)$ **7**. min of -4 at $(-1,-2)$ **8**. min of -10 at $(4,1)$

9. saddle point at $(5,2)$ **10**. max of 9 at $(0,1)$

11. min of 3 at $(1,1)$, and saddle point at $(0,0)$

12. max of 10 at $(1,-1)$, and saddle point at $(0,0)$

19. $x = 300,\ y = 200$ **20**. $x = 100,\ y = 400$ **21**. $x = 2000,\ y = 1000$

22. $x = 2000,\ y = 3000$