MULTIPLE CHOICE SECTION - EACH PROBLEM IS WORTH 5 POINTS
(In Problems 1-12 mark the correct answer on the front cover)

1. If \$12,000 is deposited in an account earning 8% continuously compounded interest, how much money is in the account 8 years later?

(a) $8000e^{-.64}$ (b) $12,000e^{.8}$ (c) $8000e^{.96}$ (d) $12,000e^{.64}$ (e) $12,000e^{-.8}$

2. Find the constant k such that the function $y = x^4 + k$ is a solution to the differential equation $xy' - 4y = -16$.

(a) -6 (b) 6 (c) 4 (d) -2 (e) -4

3. A certain investment pays 5% interest compounded continuously. Suppose that deposits are made into the account in the following manner. An initial deposit of \$5000 is made. After 2 years, additional money is deposited continuously into the account at the rate of \$1000 per year. This lasts for 4 years. How much money is in the account 6 years after the initial investment?

(a) $5000e^{.3} + \int_0^6 1000e^{.05(6-t)}dt$ (b) $\int_0^6 5000e^{.05(6-t)}dt$

(c) $5000e^{.3} + \int_0^4 1000e^{.05(4-t)}dt$ (d) $e^{.1} \int_0^4 1000e^{.05(4-t)}dt$

(e) $5000e^{.3} \int_0^4 1000e^{.05(4-t)}dt$

4. Widgets-R-Us makes a profit of \$500,000 per day when they produce (and sell) 10 widgets. The company finance director estimates that the marginal profit function for widget production is $-2x + 20$ in *thousands* of dollars per day. What will be the change in profit (in thousands of dollars) if the company increases production from 10 to 20 widgets?

(a) $-\$100$ (b) \$600 (c) \$100 (d) $-\$400$ (e) \$400

5. Compute the improper integral $\int_{-\infty}^{-1} \frac{1}{x^3} dx$.

(a) -1 (b) $-1/2$ (c) $1/2$ (d) 1 (e) the integral diverges

6. Compute the average value of $f(x) = 3x^2$ on $1 \le x \le 4$.

(a) 189 (b) 63 (c) 42 (d) 21 (e) 9

7. Find the present value of a perpetual income stream flowing continuously at a rate of \$32,000 per year, and with interest compounded continuously at the rate of 8%.

(a) \$400,000 (b) \$380,000 (c) \$390,000 (d) \$410,000 (e) \$420,000

8. Use the trapezoidal rule with 3 subintervals to approximate the value of $\int_0^1 e^{x^2} dx$.

(a) $\frac{1}{3}(1 + 2e^{\frac{1}{9}} + 2e^{\frac{4}{9}} + e)$ (b) $\frac{1}{3}(e^{\frac{1}{9}} + e^{\frac{4}{9}} + e)$ (c) $\frac{1}{2}(1 + 2e^{\frac{1}{9}} + 2e^{\frac{4}{9}} + e)$

(d) $\frac{1}{3}(1 + e^{\frac{1}{9}} + e^{\frac{4}{9}})$ (e) $\frac{1}{6}(1 + 2e^{\frac{1}{9}} + 2e^{\frac{4}{9}} + e)$

9. After graduating from ND, Jane Q. Student begins to pay back her \$60,000 student loan. She pays money continuously at the rate of \$4000 per year and is charged 4% interest (continuously compounded). Let $M(t)$ be the amount of money she still owes on the loan. Set up an initial value problem which models this situation.

(a) $\dfrac{dM}{dt} = .04M + 8000, \quad M(10) = 0$ (b) $\dfrac{dM}{dt} = .04M - 4000, \quad M(0) = 60,000$

(c) $\dfrac{dM}{dt} = .04M - 60,000, M(0) = 4000$ (d) $\dfrac{dM}{dt} = .04M + 10,000, M(10) = 0$

(e) $\dfrac{dM}{dt} = .04M + 4000, M(0) = 60,00$

10. A person deposits money into an account continuously at the rate of $s(t) = 1000 + 100t$ dollars per year for 12 years. Assuming that the account pays 5% continuously compounded interest, what is the present value of this investment?

(a) $\int_0^{12} 1000e^{.05(12-t)}dt$

(b) $\int_0^{12}(1000t + 50t^2)e^{-.05t}dt$

(c) $\int_0^{12}(1000 + 100t)e^{.05(12-t)}dt$

(d) $\int_0^{12}(1000 + 100t)e^{-.05t}dt$

(e) $\int_0^{12} 1000e^{-.05t}dt$

11. To solve the differential equation $\frac{dp}{dt} = 3p - 4p^2$, which of the following pairs of integrals would you need to compute?

(a) $\displaystyle\int \ln(3p - 4p^2)dp$ and $\displaystyle\int 1dt$

(b) $\displaystyle\int (3p - 4p^2)dp$ and $\displaystyle\int 1dt$

(c) $\displaystyle\int \frac{1}{3p - 4p^2}dp$ and $\displaystyle\int tdt$

(d) $\displaystyle\int \frac{1}{3p}dp$ and $\displaystyle\int 4t^2dt$

(e) $\displaystyle\int \frac{1}{3p - 4p^2}dp$ and $\displaystyle\int 1dt$

12. Your little sister opens her first bank account with $100. The account pays 6% interest compounded continuously and she wants to know how many years it will be until her money doubles?

(a) $\frac{10}{3}$

(b) $\frac{\ln 2}{.06}$

(c) $100e^{.6}$

(d) $\frac{\ln(.5)}{.06}$

(e) $100e^{-.6}$

PARTIAL CREDIT SECTION
(In Problems 13-15 show all work on the paper)

13. (Partial credit, 20 points) The supply and demand curves for a certain item are

$$p = S(q) = q + 1 \qquad \text{and} \qquad p = D(q) = \frac{64}{q+1}.$$

(a) Find the equilibrium price and equilibrium quantity.

(b) Set up but do **not** evaluate the expression which gives the consumer surplus.

(c) Set up but do **not** evaluate the expression which gives the producer surplus.

(d) Give a rough sketch of the graphs of the demand and supply curves, indicating the equilibrium price and quantity, and the regions whose area represent the consumer and producer surplus.

14. (Partial credit, 10 points) Consider the differential equation $\dfrac{dy}{dt} = 3t^2y^2$.

(a) Find the general solution.

(b) Find the particular solution which satisfies $y(1) = -1/8$.

15. (Partial credit, 10 points) Consider the Logistic differential equation $\dfrac{dp}{dt} = 2p - 6p^2$. To solve the differential equation, from the method of separation of variables, we get the equation

$$\ln\left|\frac{p}{1-3p}\right| = 2t + C.$$

Find the general solution to the differential equation by solving this equation for p in terms of t.