Name: ―――――――――   **Instructor:** Viorel Nitica

Math 108.02, Calculus II for Business
Fall Semester 1998
Exam 1
Friday, September 18, 3:00-3:55 PM

This Examination contains **13** problems, worth a total of 100 points, on (10) sheets of paper including the front cover. The first **9** problems are multiple choice with no partial credit, and each is worth 5 points. Record your answers to these problems by placing an × through one letter for each problem below:

1. a   b   c   d   e   6. a   b   c   d   e

2. a   b   c   d   e   7. a   b   c   d   e

3. a   b   c   d   e   8. a   b   c   d   e

4. a   b   c   d   e   9. a   b   c   d   e

5. a   b   c   d   e

The last **four** problems are partial credit problems worth a total of 55 points. For these problems, **show** your computations and **clearly** mark your answers on the page. Books and notes are not allowed. You may use your calculator.

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

―――――――――――――――――

GOOD LUCK

1. (5 pts.) The rate of change of an investment is $s(t) = 500e^{0.4t}$ dollars per year. If $2,000$ was initially invested, find the value of the investment after t years.

    a) $V(t) = 1,250e^{0.4t} + 750$;

    b) $V(t) = 2,000e^{0.4t}$;

    c) $V(t) = 1,250e^{0.4t} - 750$;

    d) $V(t) = 400e^{0.4t} + 2000$;

    e) $V(t) = 400e^{0.4t} + 1,600$.

2. (5 pts.) Consider the initial value problem:

$$\frac{dy}{dx} = 3x^{1/2} + 2x, \quad y(4) = 33.$$

Find $y(1)$.

    a) -5;   (b) 4;   (c) 3;   (d) 2;   (e) 0.

3. (5 pts.) The substitution $u = x + 3$ changes the integral

$$\int x\sqrt{x+3} \, dx$$

into which one of the following integrals?

a) $\int (u^{5/2} - 3u^{1/2}) \, du$;

b) $\int (u^{3/2} + 3u) \, du$;

c) $\int \sqrt{u} \, du$;

d) $\int (u^{3/2} - 3u^{1/2}) \, du$;

e) $\int (u^2 + 3u) \, du$.

4. (5 pts.) Compute the definite integral: $\displaystyle\int_0^1 \frac{2x+3}{x^2+3x+1} \, dx$.

a) $\ln 5 - 1$; b) $\ln 5$; c) 0; d) $\dfrac{-24}{25}$; e) $\dfrac{-51}{25}$.

5. (5 pts.) Which one of the following methods would you use to find the indefinite integral $\int x^2 \ln(4x) \, dx$?

    a) Substitution with $u = 2x$;

    b) Integration by parts with $u = x^2$, $dv = \ln(4x)dx$;

    c) Integration by parts with $u = \ln(4x)$, $dv = x^2 dx$;

    d) Substitution with $u = x^2$;

    e) Partial fractions.

6. (5 pts.) Using the method of integration by parts with $u = x^3$, $v' = e^x$, the integral

$$\int x^3 e^x \, dx$$

becomes which of the following expressions?

    a) $x^3 e^x - \int 3x^2 e^x \, dx$;

    b) $3x^2 e^x - 3e^x \, dx$;

    c) $3e^x + x^2 e^x \, dx$;

    d) $\int x^3 e^x \, dx - 3x^2 e^x \, dx$;

    e) $x^3 e^x - \int x^2 e^x \, dx$;

7. (5 pts.) Find the value of the definite integral from $x = -4$ to $x = 4$ of the function whose graph is shown in the following picture.

a) -12; b) -8; c) 8; d) -4; e) 4.

8. (5 pts.) Suppose the function $f(x)$ takes the following values

Estimate $\displaystyle\int_{-1}^{1} f(x) \, dx$ by computing the Riemann sum with 4 subintervals and left end-points.

a) 1; b) 1/2; c) 0; d) -1; e) -1/2.

9. (5 pts.) Which of the following expressions is a Riemann sum of the integral

$$\int_0^1 e^{x^2} \, dx?$$

a) $\dfrac{1}{3}(1 + 2e^{1/9} + 2e^{4/9} + e)$;

b) $\dfrac{1}{2}(e^{1/9} + e^{4/9} + e)$;

c) $\dfrac{1}{3}(1 + 2e^{1/9} + e^{4/9} + 2e)$;

d) $\dfrac{1}{3}(1 + e^{1/9} + e^{4/9})$;

e) $\dfrac{1}{3}(1 + e^{1/9} + e^{4/9} + e)$.

10. (15 pts.) a)Compute the definite integral

$$\int_0^1 \frac{1}{1+x}\, dx.$$

b) Compute the Riemann sum for the integral above using 4 subintervals and right end-points.

c) Using a) and b), find an approximation for $\ln 2$. Explain your answer.

11. (15 pts.) The marginal revenue for producing and selling x pairs of boots is given by $MR = -3x + 1500$. When 100 pairs of boots are sold, the revenue of the company is $\$40,000$.

a) Determine how many pairs of boots should be sold by the company in order to maximize its revenue. Explain your answer.

b) Find the revenue function.

c) Compute the maximum possible revenue.

12. (10 pts.) a) Find the partial fraction decomposition of

$$\frac{1}{x^2 - 5x + 4}.$$

That is find the numbers A and B such that

$$\frac{1}{x^2 - 5x + 4} = \frac{A}{x - 4} + \frac{B}{x - 1}.$$

b) Compute

$$\int_6^8 \frac{1/2}{x - 3}.$$

13. (15 pts.) Consider the functions $f(x) = x^2 - 1$ and $g(x) = x + 1$.

a) Find the x-values of the points where the graphs of $f(x)$ and $g(x)$ intersect.

b) Draw the graphs of the functions $f(x)$ and $g(x)$ using the same coordinate system.

c) Write the definite integral that gives the area of the region enclosed by the graphs of $f(x)$ and $g(x)$. Do not evaluate the integral.