Name: ____________________ **Instructor:** Viorel Nitica

Math 108.02, Calculus II for Business
Fall Semester 1998
Exam 3
Wednesday, November 18, 3:00-3:55 PM

This Examination contains **15** problems, worth a total of 100 points, on (10) sheets of paper including the front cover. The first **11** problems are multiple choice with no partial credit, and each is worth 5 points. Record your answers to these problems by placing an $\times$ through one letter for each problem below:

1.	a	b	c	d	e	7.	a	b	c	d	e
2.	a	b	c	d	e	8.	a	b	c	d	e
3.	a	b	c	d	e	9.	a	b	c	d	e
4.	a	b	c	d	e	10.	a	b	c	d	e
5.	a	b	c	d	e	11.	a	b	c	d	e
6.	a	b	c	d	e						

The last **four** problems are partial credit problems worth a total of 45 points. For these problems, **show** your computations and **clearly** mark your answers on the page. Books and notes are not allowed. You may use your calculator.

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

____

GOOD LUCK

1. (5 pts.) Find the linear function f(x,y) which satisfies $f(0,1) = 5, f(1,0) = 3, f(1,-1) = 0$.

a) $f(x,y) = 3x + 5y;$

b) $f(x,y) = 3y + 2x + 2;$

c) $f(x,y) = x + 3y + 2;$

d) $f(x,y) = x + 3y - 2;$

e) $f(x,y) = 3x + 2y - 2.$

2. (5 pts.) Assume that x, y, λ are the dimensions of a box. Solve the following system of equations for x, y, λ:

$$xy + y\lambda = 15$$
$$xy + x\lambda = 7$$
$$\lambda x + \lambda y = 16.$$

a) (2,3,4); (b) (1,2,3); (c) (1,3,4); (d) (1,2,4); (e) (2,3,2).

3. (5 pts.) Write the equation of the plane that contains the points $(0, 3, 0)$, $(3, 0, 0)$, $(0, 0, -3)$.

a) $z = x + y + 3$

b) $z = -x - y + 3$

c) $z = x + y - 3$

d) $z = -3x - 3y - 3$

e) None of the above.

4. (5 pts.) Write the equation of the plane that contains the points $(3, 4, -1)$, $(2, 4, 0)$, $(3, -1, -1)$.

a) $z = -x + 2$

b) $z = -y + 2$

c) $z = 3x + 4y - 1$

d) $z = x - y$

e) None of the above.

5. (5 pts.) For which value(s) of k will the following system of equations have infinitely many solutions:

$$
\begin{aligned}
x + y + z &= 3 \\
x + 2y + 2z &= 4 \\
2x + 2y + 2kz &= 6
\end{aligned}
$$

a) k=0,-1;

b) k=1;

c) k=-1;

d) k=2;

e) None of the above.

6. (5 pts.) Find the equation of the tangent plane to the graph of the function $f(x,y) = x^2 + ye^x$ at (1,1).

a) $z = x - 2y - 1$

b) $z = 3x + y - 2$

c) $z = 3x - y + 1$

d) $z = x - 2y + 1$

e) None of the above.

7. (5 pts.) Which of the following is the function that must be minimized in order to find the Least Squares Fit straight line ($y = ax + b$) through the three points $(0, 1), (1, 3), (2, 4)$?

a) $E(a, b) = (b - 1)^2 + (a + b - 3)^2 + (2a + b - 4)^2$;

b) $E(a, b) = (a + b - 1)^2 + (2a + b - 3)^2 + (2a + b - 4)^2$;

c) $E(a, b) = (b - 1)^2 + (2a + b - 2)^2 + (3a + b - 3)^2$;

d) $E(a, b) = (2a + b)^2 + (a + b - 1)^2 + (2a + b - 3)^2$;

e) $E(a, b) = (b - 2)^2 + (a + b - 3)^2 + (2a + b - 3)^2$.

8. (5 pts.) Suppose that $f(x, y) = xe^y + ye^x + ln(xy)$. Find $\dfrac{\partial^2 f}{\partial x^2}$.

a) $\dfrac{\partial^2 f}{\partial x^2} = ye^x - \dfrac{1}{x^2}$

b) $\dfrac{\partial^2 f}{\partial x^2} = xe^y - \dfrac{1}{y^2}$

c) $\dfrac{\partial^2 f}{\partial x^2} = ye^x + \dfrac{1}{x}$

d) $\dfrac{\partial^2 f}{\partial x^2} = xe^y + \dfrac{1}{y}$

e) None of the above.

9. (5 pts.) A factory makes two kinds of widgets:α-type and β-type. The profit from selling x widgets of α-type, and y widgets of β-type is a function $P(x, y)$. Assume that $P(100, 200) = \$80,000$, $\frac{\partial P}{\partial x}(100, 200) = 500$, $\frac{\partial P}{\partial y}(100, 200) = 1000$. Estimate the profit of the company if it sells 80 widgets of α-type and 180 widgets of β-type.

a) 80,000;

b) 90,000;

c) 100,000;

d) 110,000;

e) 120,000.

10. (5 pts.) A function $f(x, y)$ satisfies $\frac{\partial f}{\partial x} = 0 = \frac{\partial f}{\partial y}$ at the point $(-3, 4)$. Suppose also that $\frac{\partial^2 f}{\partial x^2} = 3$, $\frac{\partial^2 f}{\partial y^2} = 4$, $\frac{\partial^2 f}{\partial x \partial y} = 3$ at the point $(-3, 4)$. Which of the following is true?

a) $(-3, 4)$ is a relative maximum point;

b) $(-3, 4)$ is a saddle point;

c) $(-3, 4)$ is a relative minimum point;

d) $(-3, 4)$ is not a critical point;

e) $(-3, 4)$ is a critical point, but the second derivative test is inconclusive.

11. (5 pts.) At which point is the x-slope of the surface $z = x^2 + xy$ **not** equal to 3?

a) (1,1) b) (0,3) c) (2,-1) d) (-2,4) e) (3,-3).

12. (5 pts.) Four of the vertices of a cube are $(1,1,1)$, $(4,4,4)$, $(1,1,4)$, and $(4,1,1)$. Find the coordinates of the center of the cube.

13. (10 pts.) Consider the function $f(x, y) = 2x^2 + 2y^2$. Sketch the level curves for $f(x, y)$ at the values $z = -4, 0, 8, 32$ on the following graph. Label each curve. If a level curve does not exist, say so.

14. (15 pts.) The volume of a cylindrical beverage can **with open top** is 340 cubic centimeters.

a) Assume that you want to find the radius of the basis and the height of the can which minimize its surface area. Write the related constrained optimization problem.

b) Using the method of Lagrange multipliers, set up a system of equations whose solution gives the possible points at which the minimum could occur.

c) Solve the system of equations.

15 (15 pts.) A firm manufactures and sells two products, X and Y, that sells for \$10 and \$13 each, respectively. The cost of producing x units of X, and y units of Y is

$$C(x, y) = 100 - 2x + 5y + 0.01(2x^2 + xy + 2y^2).$$

Find the values of x and y that maximize the firm's profit.