Name: ____________________ **Instructor:** Viorel Nitica

Math 108.02, Calculus II for Business
Fall Semester 1998
Exam 3
Wednesday, November 18, 3:00-3:55 PM

This Examination contains **15** problems, worth a total of 100 points, on (10) sheets of paper including the front cover. The first **11** problems are multiple choice with no partial credit, and each is worth 5 points. Record your answers to these problems by placing an $\times$ through one letter for each problem below:

1.	a	b	c	d	e	7.	a	b	c	d	e
2.	a	b	c	d	e	8.	a	b	c	d	e
3.	a	b	c	d	e	9.	a	b	c	d	e
4.	a	b	c	d	e	10.	a	b	c	d	e
5.	a	b	c	d	e	11.	a	b	c	d	e
6.	a	b	c	d	e						

The last **four** problems are partial credit problems worth a total of 45 points. For these problems, **show** your computations and **clearly** mark your answers on the page. Books and notes are not allowed. You may use your calculator.

Sign the pledge: "On my honor, I have neither given nor received unauthorized aid on this Exam":

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GOOD LUCK

1. (5 pts.) The rate of change of an investment is $R(t) = 20t + 5$. If \$1000 was initially invested, find the value after 5 years.

a) 1325;

b) 1300;

c) 1025;

d) 1275;

e) 1225.

2. (5 pts.) Solve the initial value problem

$$\frac{dy}{dx} = 3x^2 + 1, \quad y(1) = 2.$$

a) $y = x^3 + x + 2$;

(b) $y = 3x^3 + x + 2$;

(c) $y = x^3 + x$;

(d) $y = 3x^3 + x$;

(e) $y = x^3 + x - 2$.

3. (5 pts.) Write the equation of the plane that contains the points $(0, 0, 3)$, $(2, 0, 1)$, $(0, 2, 1)$.

a) $z = x + y + 3$

b) $z = -x - y + 3$

c) $z = x + y - 3$

d) $z = -3x - 3y - 3$

e) None of the above.

4. (5 pts.) Which of the following methods would you use to compute

$$\int xe^{2x} dx.$$

a) Integration by parts with $u = e^{x^2}$ and $v' = x$.

b) Integration by parts with $u = x$ and $v' = e^{x^2}$.

c) Partial fractions.

d) Substitution with $u = x^2$.

e) Direct integration using formulas.

5. (5 pts.) For which value(s) of k will the following system of equations have no solution:

$$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y + 2z &= 5 \\ 2x + 4y + kz &= 5 \end{aligned}$$

a) k=0,-1;

b) k=1;

c) k=-1;

d) k=2;

e) None of the above.

6. (5 pts.) Find the equation of the tangent plane to the graph of the function $f(x, y) = x^2 - y$ at (1,0).

a) $z = 2x - y$

b) $z = 2x - y + 1$

c) $z = 2x - y - 1$

d) $z = x - 2y + 1$

e) $z = x - 2y$.

7. (5 pts.) Compute $\int x \ln x dx$ using integration by parts.

a) $x^2 \ln x - \frac{x^2}{2} + C$;

b) $\frac{x^2}{2} \ln x + \frac{x^2}{4} + C$;

c) $\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$;

d) $\frac{x^2}{2} - \frac{x^2}{4} \ln x + C$;

e) None of the above.

8. (5 pts.) Suppose the function $f(x)$ takes the following values:

Estimate $\int_{-1}^{1} f(x)dx$ by computing the Riemann sum with 4 subintervals and left endpoints.

a) 1

b) 1.9

c) 2

d) 1.1

e) 1.45

9. Find the partial fraction decomposition of

$$\frac{1}{x^2 - 6x + 5}.$$

That is find the numbers A and B such that:

$$\frac{1}{x^2 - 6x + 5} = \frac{A}{x-5} + \frac{B}{x-1}.$$

a) $A = 1, B = -1$

b) $A = \frac{-1}{4}, B = \frac{1}{4}$

c) $A = \frac{1}{4}, B = \frac{-1}{4}$

d) $A = \frac{-1}{3}, B = \frac{1}{3}$

e) $A = \frac{1}{3}, B = \frac{-1}{3}$

10. (5 pts.) Find the solution of the following initial value problem:

$$\frac{dy}{dt} = 4y^2t^3, \quad y(0) = -1.$$

a) $y = -\frac{1}{4t^4 + 1}$;

b) $y = -\frac{1}{t^4 + 1}$;

c) $y = \frac{1}{4t^4 + 1}$;

d) $y = \frac{1}{t^4 + 1}$;

e) None of the above.

11. (5 pts.) Compute the improper integral:

$$\int_1^\infty \frac{2x}{(x^2+1)^2}\,dx.$$

a) 1/2;

b) 1/4;

c) 1;

d) 0;

e) None of the above.

12. (5 pts.) Find the present value of a perpetual income stream flowing continuously at a rate of $\$20,000$ per year, with interest rate compounded continuously at 8%.

a) 140,000

b) 160,000

c) 180,000

d) 200,000

e) 14,000

13. (5 pts.) Solve the following equation for p in terms of t.

$$\frac{1}{2}\ln\left(\frac{p-1}{p}\right) = 2t+1.$$

a) $p = \dfrac{1}{1-e^2e^{4t}}$;

b) $p = \dfrac{1}{1+e^{4t}}$;

c) $p = \dfrac{1}{1-e^{4t}}$;

d) $p = \dfrac{1}{1+e^2e^{4t}}$;

e) $p = e^{-2-4t}$.

14. (5 pts.) The level curve for the function $f(x,y) = (2x^2+2y^2)^2$ with height $z=1$ is which of the following?

a) a line;

b) a circle;

c) a point;

d) a parabola;

e) does not exist.

15. An individual opens an account with initial amout of $\$100,000$, and then makes continuous withdrawals at the rate of $\$2,000$ per year. We assume an interst rate of $\%5$ compounded continuously. Find the differential equation that models the amount of money $M(t)$ in the account.

a) $\dfrac{dM}{dt} = .05M + 2000,\ M(0) = 100,000$

b) $\dfrac{dM}{dt} = -.05M - 2000,\ M(0) = 100,000$

c) $\dfrac{dM}{dt} = .05M + 2000,\ M(0) = 98,000$

d) $\dfrac{dM}{dt} = -.05M + 2000,\ M(0) = 98,000$

e) $\dfrac{dM}{dt} = 2000M - 50000,\ M(0) = 100,000$

16. According to the method of Lagrange multipliers, at which of the following points (x, y) could the function $f(x, y) = 2x^2 - y^2$ possibly obtain a maximum value subject to constraint $x + y = 2$.

a) $(-2, 4)$

b) $(2, -4)$

c) $(0, -2)$

d) $(1, -2)$

e) $(-1, 2)$

17. A function $f(x, y)$ satisfies $\frac{\partial f}{\partial x} = 0 = \frac{\partial f}{\partial y}$ at the point $(0, 1)$. Suppose also that $\frac{\partial^2 f}{\partial x^2} = 1$, $\frac{\partial^2 f}{\partial y^2} = 4$, $\frac{\partial^2 f}{\partial x \partial y} = 5$ at the point $(0, 1)$. Which of the following is true?

a) (0,1) is a relative maximum point

b) (0,1) is a saddle point

c) (0,1) is a relative minimum point

d) (0,1) is not a critical point

e) (0,1) is a critical point, but the second derivative is inconclusive

18. Assume that $f(1, 2) = 4$, $\frac{\partial f}{\partial x}(1, 2) = 2$, $\frac{\partial f}{\partial y}(1, 2) = 3$. Find an aprroximation for $f(.8, 2.1)$.

a) 4.1

b) 3.9

c) 4.2

d) 3.8

e) 4.3

19. For a certain item the demand curve is $D(q) = -2q + 12$, and the supply curve is $S(q) = 3q + 2$.

a) Find the equilibrium price and the equilibrium quantity.

b) Find the consumer surplus.

c) Find the producer surplus.

20. Consider the functions $f(x) = 2x, g(x) = x^2 - x$.

a) Find the values of the points where the graph of $f(x)$ and $g(x)$ intersect.

b) Draw the graphs of $f(x)$ and $g(x)$ using the same coordinate system. Shade the region enclosed by the graphs of $f(x)$ and $g(x)$.

c) Write the definite integral that gives the area of the region enclosed by the graphs of $f(x)$ and $g(x)$.