Math 108
Exam I

January ??, 1998

**1.** A factory installs new and more efficient machines that are expected to generate savings at the rate of $600,000e^{-0.3t}$ dollars per year, after $t$ years in operation. If the cost of the machines is \$1,000,000, find the time needed for the machines to pay for their cost.

**Solution.** Let $S = S(t)$ be the savings after $t$ years. Then $S'(t) = 600,000e^{-0.3t}$ and $S(0) = 0$. Thus $S(t) = \int ?600,000e^{-0.3t}dt$ or $s(t) = \frac{600,000}{-0.3}\ e^{-0.3t} + c = -2,000,000e^{-0.3t} + c$.

Since $0 = S(0) = -2,000,000 + c$ we have $c = 2,000,000$. Therefore $S(t) = 2,000,000(1 - e^{-0.3t})$. To find the time needed for the machines to pay for their cost, we solve the equation $2,000,000(1 - e^{-0.3t}) = 1,000,000$, or $1 - e^{-0.3t} = \frac{1}{2}$ or $e^{-0.3t} = 0.5$ or $-0.3t = \ell n 0.5$ or $t = -\frac{\ell n 0.5}{0.3} \simeq 2.3$ years.

**2.** The marginal profit function of a company for producing and selling $x$ units of a product is given by $MP = -0.6x + 1,200$ in dollars. When 500 units are produced and sold, the company's profit is 25,000. Find:

a. The profit function

b. The production level which maximixes the profit.

**Solution.**

a. Let $P(x)$ be the profit function. Then $P' = -0.6x + 1,200$, and $P(500) = 25,000$. Thus $P = \int(-0.6x + 1,200)dx$ or $P(x) = -0.3x^2 + 1,200x + c$. Since $25,000 = P(500) = -0.3 \cdot 250,000 + 1,200 \cdot 500 + c$ or $c = -500,000$. Thus
$$P(x) = -0.3x^2 + 1,200x - 500,000.$$

b. The profit is maximized when $MP = 0$ or $-0.6x + 1,200 = 0$ or $x = -1,200/0.6$ or $x = 2,000$.

**3.** Consider the following situation. You are hired to advise a company for maximizing its profit. You find that it produces and sells daily 3,000 units of a certain product for the profit of \$100,000. You also estimate the marginal profit function to be $MP = -0.2x + 800$, where $x$ is the number of units produced and sold. Compute the amount of units which the company needs to produce and sell to maximize its profit. Also compute the maximum profit.