Math 108
Exam I

February 3, 1998

3. To find the points where the two curves intersect we set $4 - x^2 = -2x + 1$. Then we obtain $x^2 - 2x - 3 = 0$ or $(x+1)(x-3) = 0$. Thus the two curves intersect for $x = -1$ and $x = -3$. Their graphs look like the following picture.

Insert Graph

The area of the enclosed region is given by

$$\int_{-1}^{3} [4 - x^2 - (-2x+1)]dx = \int_{-1}^{3} (-x^2 + 2x + 3)dx =$$

$$= \left(-\frac{x^3}{3} + x^2 + 3x\right)\Big|_{-1}^{3} = (-9 + 9 + 9) - \left(\frac{1}{3} + 1 - 3\right) = \boxed{\frac{32}{3}}.$$

4. d 14/3

We have area

$$= \int_{1}^{4} x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Big|_{1}^{4} = \frac{2}{3} x^{\frac{3}{2}}\Big|_{1}^{4} =$$

$$= \frac{2}{3}\left[4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] = \frac{2}{3}[8-1) = \boxed{\frac{14}{3}}$$

5.a 11/8

The four subintervals are $\left[0, \frac{1}{4}\right], \left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{3}{4}, 1\right]$, and $\Delta x = \frac{1}{4}$. Thus the desired Riemann sum is $\left(2 - \frac{1}{4}\right) \cdot \frac{1}{4} + \left(2 - \frac{1}{2}\right) \cdot \frac{1}{4} + \left(2 - \frac{3}{4}\right) \cdot \frac{1}{4} + (2-1) \cdot \frac{1}{4} = \boxed{\frac{11}{8}}$.

6.b 3

We have $y(x) = \int (2x^3 - x + 1)dx = \frac{1}{2}x^4 - \frac{1}{2}x^2 + x + c$.. Since $0 = y(1) = \frac{1}{2} - \frac{1}{2} + 1 + c$ we have $c = -1$. Thus $y(x) = \frac{1}{4}x^4 - \frac{1}{2}x^2 + x - 1$ and $F(2) = y(2) = \frac{1}{4} \cdot 2^4 - \frac{1}{2} \cdot 2^2 + 2 - 1 = \boxed{3}$.

7.e $\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9}$

The five equal subintervals are $\left[1, \frac{6}{5}\right], \left[\frac{6}{5}, \frac{7}{5}\right], \left[\frac{7}{5}, \frac{8}{5}\right] \left[\frac{8}{5}, \frac{9}{5}\right], \left[\frac{9}{5}, 2\right]$, and $\Delta x = \frac{1}{5}$. Thus the Riemann sum corresponding to the left-hand endpoints is

$$1 \cdot \frac{1}{5} + \frac{5}{6} \cdot \frac{1}{5} + \frac{5}{7} \cdot \frac{1}{5} + \frac{5}{8} \cdot \frac{1}{5} + \frac{5}{9} \cdot \frac{1}{5} = \boxed{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9}.}$$

8.d $\dfrac{x^2}{2}\ln x - \dfrac{x^2}{4} + C$

We apply the IBP formula $\int udv = uv - \int vdu$ with $u = \ell nx$ and $dv = xdx$. Then $du = \frac{1}{x}dx$ and $v = \frac{x^2}{2}$.Thus we obtain

$$\int x\ell nxdx = \frac{x^2}{2}\ell nx - \int \frac{x^2}{2}\frac{1}{x}dx = \frac{x^2}{2}\ell nx - \frac{1}{2}\int xdx = \boxed{\frac{x^2}{2}\ell nx - \frac{x^2}{4} + c}.$$

9.b $2\int e^u du$

If we let $u = \sqrt{x} = x^{\frac{1}{2}}$ then $du = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2}\frac{dx}{\sqrt{x}}$. Therefore

$$\int \frac{1}{\sqrt{x}}e^{\sqrt{x}}dx = \boxed{2\int e^u du.}$$

10.c $\frac{1}{1+t^2}$

We have $F(t) = \int_0^t \frac{1}{1+x^2}\, dt$ and by the FTC $F'(t) = \frac{1}{1+t^2}$.

11.b -8

The definite integral is equal to $-8 - 2 + 2 = -8$.

12.d 3/20

Let $u = x^2 + 1$. Then $du = 2xdx$ and the integral becomes

$$\frac{1}{2}\int_2^5 u^{-2}du = \frac{1}{2}\frac{u^{-1}}{-1}\Big|_2^5 = -\frac{1}{2}\left(\frac{1}{5} - \frac{1}{2}\right) = \frac{3}{20}.$$

13.e 1

Let $u = x$ and $dv = e^x dx$. Then $du = dx$ and $v = e^x$. By applying the IBP formula we obtain

$$\int_0^1 xe^x dx = xe^x\Big|_0^1 - \int_0^1 e^x dx = e - e^x\Big|_0^1 = e - e + 1 = 1.$$