Math 108
Exam 2 Solutions

1.a. Intersection of Demand and Supply Curves $\frac{25}{q+1} = q+1$ therefore $(q+1)^2 = 25, q+1 = 5, q_e = 4$ and $p_e? = 5$.

1.b.

$$\begin{aligned} CS = \int_0^4 \frac{25}{q+1} dq - (4)(5) &= 25ln(q+1)\Big|_0^4 - 20 \\ &= 25ln5 - 20 \end{aligned}$$

1.c. $PS = (4)(5) - \int_0^4 (q+1) = 20 - \frac{(q+1)^2}{2}\Big|_0^4 = 20 - \left(\frac{25}{2} - 0\right) = \frac{15}{2}$

2.a. $\int \frac{dy}{4-y} = \int dt +$ const. $= t +$ const.

$$\left\| \begin{array}{ll} n & = 4-y \\ dn & = -dy \end{array} \right.$$

$$-\int \frac{dn}{n} = -ln|n| + \text{ const.} = -ln|4-y| + \text{const.}$$

$$\begin{aligned} ln|4-y| &= -t + c \\ |4-y| &= e^{-t+c} = Ke^{-t} \\ 4-y &= (\pm K)e^{-t} \\ 4-y &= ke^{-t}. \end{aligned}$$

2.b . Equilibrium Solution $y = 4$

2.c.

$$\begin{aligned} y > 4 &\implies 4-y < 0 \implies \frac{dy}{dt} < 0 \implies y \text{ decreasing} \\ y < 4 &\implies 4-y > 0 \implies \frac{dy}{dt} > 0 \implies y \text{ increasing} \end{aligned}$$

Therefore $y = 4$ is a stable solution.

3.

$$\frac{1}{x-x^2} = \frac{1}{x(1-x)} = \frac{A}{x} + \frac{B}{1-x} = \frac{A - Ax + Bx}{x(1-x)} = \frac{A + (B-A)}{n(1-n)}.$$

Therefore $A + (B-A)x = 1$ forall x

Therefore $B - A = 0$ and $A = 1$. Therefore $B = 1$.

To??

$$\begin{aligned} \int \frac{1}{x(1-x^2)}dx &= \int \frac{1}{x} \overset{dx}{+} \int \frac{1}{1-x} ax?? \\ &= ln|x| - ln|1-n| + C \\ &= ln\frac{|n|}{(n-1)} + C. \end{aligned}$$

4.b.

$$\frac{dy}{y^2} = e^t dt$$

$$-\frac{1}{y} = e^t + c$$

$$y(0) = -\frac{1}{8} \quad \frac{-1}{(\frac{-1}{8})} = 1 + c \quad \text{therefore } c = 7$$

$$\text{Therefore } y = \frac{-1}{e^t + 7}$$

5. c.

$$\begin{aligned} \frac{-2(x-60)^4}{4} + 4,000x\Big|_{60}^{70} &= \frac{-(x-60)^4}{2} + 4000x\Big|_{60}^{70} \\ &= -5000 + 280,000 - (0 + 240,000) \\ &= 35000 \end{aligned}$$

6.e.

$$\begin{aligned} \int_0^\infty \frac{1}{(x+1)^5}dx &= \lim_{b\to\infty} \int_0^b \frac{1}{(x+1)^5}dx \\ \int_0^b \frac{1}{(x+1)^5}dx &= \frac{(x+1)^{-4}}{-4}\Big|_0^b = \frac{-1}{4}\left\{\frac{1}{(b+1)^4} - \frac{1}{14}\right\} \\ \lim_{b\to\infty} \int_0^b \frac{1}{(x+1)^5}dn &= \frac{1}{4} \end{aligned}$$

7.d.

$$\frac{1}{3} \quad \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\Big|_1^4 \qquad AV = \frac{\int_1^4 \sqrt{x}\ dx}{3}$$

$$\frac{2x^{\frac{3}{2}}}{9}\Big|_1^4 = \frac{2}{9}\left\{(4)^{\frac{3}{2}} - 1^{\frac{3}{2}}\right\} = \frac{2}{9}(8-1) = \frac{14}{9}$$

8.b.

$$
\begin{aligned}
\int_0^\infty 14000e^{-.07t}dt &= \lim_{b\to\infty}\int_0^b 14000\cdot e^{-(0.7)t}dt \\
&= \lim_{b\to\infty}(14000)\frac{e^{-(0.7)t}}{(-0.7)}\bigg|_0^b \\
&= \frac{14000}{-0.7}\{0-1\} = \frac{14000}{7}\cdot 100 \\
&= 200,000
\end{aligned}
$$

9.1. We have $f(x)=\sqrt{x^2+1}, x_1=0, x_2=1, x_3=2, x_4=3, x_5=4$ and $bx=1$. Therefore

$$
\begin{aligned}
\int_0^4 \sqrt{x^2+1}\,dx &\simeq \left[\sqrt{1}+2\sqrt{1^2+1}+2\sqrt{2^2+1}+2\sqrt{3^2+1}+2??\sqrt{4^2+1}\right]\frac{1}{2} \\
&= \frac{1}{2}\left(1+2\sqrt{2}+2\sqrt{5}+2\sqrt{10}+\sqrt{15}\right) \simeq 9.374.?????9.373??
\end{aligned}
$$

10. We have $\dfrac{dM}{dt} = 0.1M - 50,000.$

11.d. For $y = x^3 + k$ to be a solution we must have $x(x^3+k)^1 - 3(x^3+k) = 15$, or $x\cdot 3x^2 - 3x^3 - 3k = 15$, or $-3k = 15$ or $\boxed{k=-5}$.

12. $\dfrac{dW}{dt} = 1,000,000e^{0.1t}$

$$
\begin{aligned}
\text{Change in } W &= \int_0^{10} 1,000,000e^{0.1t}df \\
&= 1,000,000\frac{e^{(0.1)t}}{(0.1)}\bigg|_0^{10} \\
&= 10,000,000(e-1). \\
&- 17,000.000.
\end{aligned}
$$