Math 108
Final Solutions

1.

(a.) We solve $D(q) = S(q)$ or $\frac{125}{q+1} = (q+1)^2$ or $(q+1)^3 = 125$ or $q+1 = 5$ or $\boxed{q_e = 4}$. Then $p_e = (4+1)^2$ or $\boxed{p_e = 25}$.

(b.) The consumer surplus is

$$CS = \int_0^4 \frac{125}{q+1} dq - 4 \cdot 25 = 125\ell n(q+1)\Big|_0^4 - 100 = \boxed{125\ell n5 - 100}.$$

(c.) The producer surplus is

$$PS = 100 - \int_0^4 (q+1)^2 dq = 100 - \frac{1}{3}(q+1)^3\Big|_0^4 = 100 - \frac{125-1}{3} = \boxed{\frac{176}{3}}.$$

2. We have

$$\frac{\partial P}{\partial x} = 10 - 0.02(2x+y) \text{ and } \frac{\partial P}{\partial y} = 16 - 0.02(x+6y).$$

To find the critical points, we solve $\frac{\partial P}{\partial x} = 0$ and $\frac{\partial P}{\partial y} = 0$. This is $10 - 0.02(2x+y) = 0$ and $16 - 0.02(x+6y) = 0$. Computing gives $2x + y = 500$ and $x + 6y = 800$. Solving gives $\boxed{x = 200 \text{ and } 6 = 100}$. To use the second derivative test we need

$$\frac{\partial^2 P}{\partial x^2} = -0.04 \frac{\partial^2 P}{\partial y^2} = -0.12, \ \frac{\partial^2 P}{\partial x \partial y} = -0.02.$$

Then

$$D = \left(\frac{\partial^2 P}{\partial x^2}\right)\left(\frac{\partial^2 P}{\partial y^2}\right) - \left(\frac{\partial P}{\partial x \partial y}\right)^2 = (-0.04)(-0.12) - (0.02)^2.$$

Since $D = 0.0044 > 0$ and $\frac{\partial^2 P}{\partial x^2} = -0.04 < 0$, the values $x = 200$ and $y = 10$ maximize the profit. Then we compute that the maximum profit is $P(200, 100) = 1,000$.

3. (**Answer:** $q_e = 5, p_e = 25$)