FINAL EXAM, Math 110, May 8, 1996 NAME ________________________________

1. For what values of x does the graph of $f(x) = 2x^3 - 3x^2 - 6x + 87$ have a horizontal tangent ?

x = , ,

2. Consider the function $f(x) = x\sqrt{1-x^2}$.
   i. What is the domain of f ?

Domain:

   ii. Compute the derivative of f. What is its domain? Find the critical numbers of f.

$f'(x) =$	Domain:

Critical numbers:

iii. Over which intervals is f increasing and over which is it decreasing? Find the local maximum and minimum values of f.

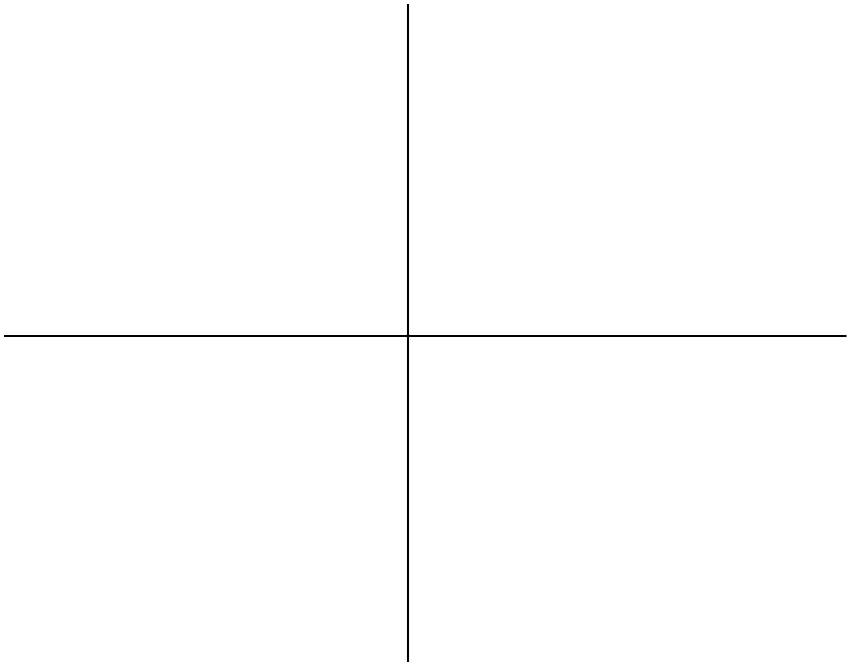
Increasing:

Decreasing:

Local Maxes:

Local Mins:

iv. Sketch the graph of f based on the information that you have developed in points (i) – (iii) (do not go to the effort of computing the second derivative).



3. Differentiate the function $y = e^{1/(1-x^2)}$.

y' =

4. Solve $\ln(x + 6) + \ln(x - 3) = \ln 5 + \ln 2$ for x

x =

5. Compute the derivatives of the following functions.

i. $g(x) = \ln (x^4 + 3x^2)$

g'(x) =

ii. $h(x) = xe^{x^2}$

h'(x) =

6. The Williamsburg Bridge spans the East River in New York City. It has four cables. Its center span is $2d = 1600$ feet, the dead load for its two decks is 19,210 pounds per foot and the live load capacity is 7,160 pounds. The sag s in the cable is 177 feet.

i. Compute the tension T_d in <u>one</u> of its cables at the tower, and the angle α that the cable makes (at the tower) with the horizontal.

$T_d =$; $\alpha =$

The Williamsburg Bridge is the only one of the East River suspension bridges for which the cables of the side span do not bear any of the load of the side span. The only purpose of each of these cables is to counterbalance T_d.

ii. If the cable over the side span makes an angle at the tower of 22.7^o with the horizontal compute the tension T in <u>one</u> of the cables of the side span.

T =

iii. Compute the <u>total</u> compression that all the cables generate in <u>one</u> tower.

C =

7. One of the waste products of a nuclear explosion is the radio-active isotope strontium–90. This isotope which behavers chemically like calcium, has a half-life of (about) 25 years. If 20 milligrams of the isotope are present in a sample now, find

   i. how much will remain in 15 years?

milligrams

   ii. in how many years will only 5 milligrams remain ?

years

8. A wood fragment tested with an accelerator mass spectrometer and is found to contain carbon-14 atoms to stable carbon atoms in a ratio of 1 to 1.596×10^{12}. Compute the age of the fragment. Assume that at the time the metabolic processes in the wood stopped, the equilibrium ratio of radioactive carbon to stable carbon was the same as it is today.

years

9. Suppose that a colony of bacteria is growing exponentially. If the number of bacteria increases from 5000 to 7000 in 13 hours, find the doubling time.

hours

10. The statistics for the population of a certain country show the following:

10 million at the beginning of 1975; an increase at a rate of 0.50 million per yr. in 1975
15 million at the beginning of 1980; an increase at a rate of 0.70 million per yr. in 1980
25 million at the beginning of 1985; an increase at a rate of 1.00 million per yr. in 1985
35 million at the beginning of 1990; an increase at a rate of 1.20 million per yr. in 1990

Let $t = 0$ correspond to the year 1975. Let $y(t)$ be the population of this country at any time $t \geq 0$ in years.

i. Verify that the population of this country satisfies the basic assumption of the logistics model by setting up an apporiate table and sketching an appropriate graph. (Work with an accuracy to within 3 decimal places.)

ii. What is the limit on the population of this country ?

Ans:

iii. Insert all relevant constants and determine $y(t)$.

y(t) =

11. Arrangements have been made with a bank for a mortgage of $100,000 at an annual interest of 7% compounded monthly. Suppose that it is taken out now and that it is to be paid back in monthly payments over the next 30 years. What will the monthly payments be ?

Ans:

12. A professor turned 30 in May of 1994 and has (since that time) been paying $1,000 per month into an account that pays at an annual rate of $r = 0.04$ compounded monthly. How much money will she have in this account when she retires at the age of 65 in May of the year 2029 ? She has decided to use this account to provide her with monthly annuity payments for the first 10 years of her retirement. What monthly payments will she receive from this account if it pays interest at an annual rate of $r = 0.05$ compounded monthly?

Ans:

13. The short run supply and demand functions for a given product in a given market are both linear. At a price of 7 dollars, the demand and supply are both 160,000 units per month. At this price the price elasticity of supply is 0.20 and the price elasticity of demand is –0.12.

    i. Determine the supply function for the product.

    S(p) =

    ii. Determine the demand function for the product.

D(p) =

iii. A cartel of consumers get together and are able to cut the demand by 20,000 units. Assuming that the short run supply situation remains the same, estimate the new equilibrium price that market forces will determine.

Ans:

FORMULAS

$T_0 = T_x \cos\theta \qquad wx = T_x \sin\theta \qquad \frac{wx}{T_0} = \tan\theta \qquad f(x) = \frac{w}{2T_0} x^2$

$T_x = w\sqrt{\frac{1}{4}\frac{d^4}{s^2} + x^2} \qquad \tan\alpha = f'(d) = \frac{2s}{d^2} \quad d = \frac{2s}{d}$

$y(t) = y_0 e^{-\lambda t} \qquad y'(t) = -\lambda y(t) \qquad 1\text{ g} = \frac{1}{m}(6.02 \infty 10^{23})$

$t = \frac{1}{\lambda} \ln\left(\frac{z(t)}{y(t)} + 1\right) \qquad t = (1.89 \times 10^9)\ln\left(\frac{9.07z(t)}{y(t)} + 1\right)$

$t = (8.26 \times 10^3)\ln\left(r_0 \frac{k}{y(t)}\right) \qquad r_0 = \frac{1}{6.463 \infty 10^{11}}$

$y(t) = y_0 e^{-\lambda t} \qquad y(t) = y_0 e^{\mu t}$

$\frac{y'}{y} = k - cy \qquad a = ck^{-1} \qquad M = \frac{1}{a} \qquad M = kc^{-1} \qquad \frac{y'}{y} = k - \frac{k}{M} y = k(1 - \frac{y}{M})$

$y(t) = \frac{Me^{kt}}{C + e^{kt}}$ or $y(t) = \frac{M}{1 + Ce^{-kt}}$ where $C = \frac{M}{y_0} - 1$

$A_p = A_0(1 + \frac{r}{n})^p$ $A(t) = A_{nt} = A_0(1 + \frac{r}{n})^{nt}$ $A(t) = A_0e^{rt}$

$S_p = \frac{12}{r} A_0 (1 + \frac{r}{12})((1+\frac{r}{12})^p - 1)$ $PV_p = \frac{12B}{r}[1 - (1 + \frac{r}{12})^{-p}]$

$PV_p = \frac{2C}{r}[1 - (1 + \frac{r}{2})^{-p}]$ $p(t) = p_0 e^{kt}$

$e_D(p) = p\frac{D'(p)}{D(p)}$ $e_S(p) = p\frac{S'(p)}{S(p)}$