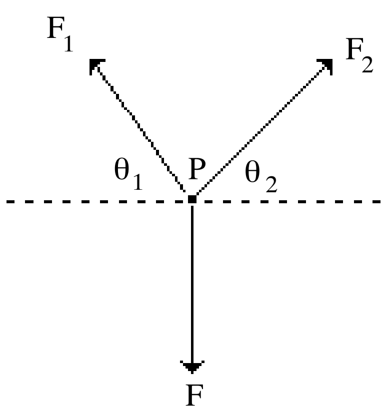
Midterm, Math 196, March 1, 1996 Name ____

1. Compute the derivative of $f(x) = (x^2 - 5x)^{\frac{1}{3}}$. After having done so, find all the critical points. Determine precisely the portions of the x-axis for which the function $f(x)$ is increasing and decreasing. Use this information to find the local maxima and minima.

2. The three forces below are in equilibrium. They all act on the point P. The dotted line is horizontal and F acts vertically. If F_1 has a magnitude of 20 pounds, and if the angles θ_1 and θ_2 are 60° and 45° respectively, determine the magnitudes of F_2 and F.



3. The Tacoma Narrows Bridge was completely rebuilt in the years 1948 to 1950. The data for the new bridge is as follows. It has a total length of 5,000 feet and a center span of 2,800 feet. Its 2 main cables support a single deck that carries 4 lanes of automobile traffic. The dead load is 8,680 pounds per foot. Assume that it is designed for a live load capacity of 4,000 pounds per foot and that thesag in the cable over the center span is 280 feet. (Based on data supplied by the Washington State Department of Transportation.) Consider one of the main cables over the center span at a point where it meets one of the towers. Compute the tension of the cable at that point and compute the angle that the cable makes with the horizontal at that point.

4. Rutherford studied the radium isotope ${}^{224}_{88}\mathrm{Ra}$. In one experiment he made the following measurements for the decay rate $y'(t)$ of the sample he was testing: at $t = 1$ day, $\frac{y'(t)}{y'(0)} = 0.88$; at $t = 2$ days, $\frac{y'(t)}{y'(0)} = 0.72$; at $t = 4$ days, $\frac{y'(t)}{y'(0)} = 0.53$; at $t = 7$ days, $\frac{y'(t)}{y'(0)} = 0.295$; at $t = 8$ days, $\frac{y'(t)}{y'(0)} = 0.252$; at $t = 11$ days, $\frac{y'(t)}{y'(0)} = 0.152$; at $t = 13$ days, $\frac{y'(t)}{y'(0)} = 0.111$. What value λ for the disintegration constant of radium–224 could Rutherford deduce from this data ?

5. Compute the following derivatives

a) $f(x) = \ln x$

$f'(x) =$ ____________________

b) $g(x) = \ln (x^4 + 3x^2)$

$g'(x) =$ _________________________

c) $f(x) = e^x$

$f'(x) =$ _____________________

d) $g(x) = e^{(x^2 - 1)}$

$g'(x) =$ __________________________

6. A radiation counter shows that a certain radioactive substance disintegrates at a rate of 8.67×10^{13} atoms per minute at a certain time and at a rate of 7.67×10^{12} atoms per minute 6 minutes later. Determine the half-life of this radioactive substance.

7. A fragment of a mineral grain is found to contain 305 parts per million rubidium–87 and 4.67 parts per million strontium–87. Given that the half-life of rubidium–87 is 4.7×10^{10} years, determine the age of the fragment.

FORMULAS

$$T_0 = T_x \cos\theta \qquad wx = T_x \sin\theta \qquad \frac{wx}{T_0} = \tan\theta \qquad f(x) = \frac{w}{2T_0}x^2$$

$$T_x = w\sqrt{\frac{1}{4}\frac{d^4}{s^2} + x^2} \qquad \tan\alpha = f'(d) = \frac{2s}{d^2} \quad d = \frac{2s}{d}$$

$$y(t) = y_0 e^{-\lambda t} \qquad y'(t) = -\lambda y(t) \qquad 1\,g = \frac{1}{m}(6.02 \infty 10^{23})$$

$$t = \frac{1}{\lambda}\ln\left(\frac{z(t)}{y(t)} + 1\right) \qquad t = (1.89 \times 10^9)\ln\left(\frac{9.07z(t)}{y(t)} + 1\right) \qquad t = (8.26 \times 10^3)\ln\left(\frac{r_0 k}{y(t)}\right)$$