Quiz, Math 100, April 3. NAME ______________________________

1. A wood fragment tested with an accelerator mass spectrometer and is found to contain carbon-14 atoms to stable carbon atoms in a ratio of 1 to 1.6174×10^{12}. Compute the age of the fragment. Assume that at the time the metabolic processes in the wood stopped, the equilibrium ratio of radioactive carbon to stable carbon was the same as it is today.

2. The statistics for the population of a certain country shows the following:

8.45 million at the beginning of 1975 and that it increased at a rate of 0.24 million per yr. in 1975
12.82 million at the beginning of 1980 and that it increased at a rate of 0.35 million per yr. in 1980
17.83 million at the beginning of 1985 and that it increased at a rate of 0.47 million per yr. in 1985
24.74 million at the beginning of 1990 and that it increased at a rate of 0.63 million per yr. in 1990
30.16 million at the beginning of 1995 and that it increased at a rate of 0.72 million per yr. in 1995

Let $t = 0$ correspond to the year 1975. Let $y(t)$ be the population of this country at any time $t \geq 0$ in years.

a) Show that the population of this country satisfies the basic assumption of the logistics model and determine the parameters y_0, k, and M.

b) What is the limit on the population of this country ?

c) After inserting all relevant constants, $y(t) =$

d) Assuming that the growth pattern observed in the period from 1975 to 1995 continues, what would you expect the country's population to be in the year 2020.

Some formulas

$$t = \frac{1}{\lambda}\ \ln\left(\frac{z(t)}{y(t)} + 1\right) \qquad t = (1.89 \times 10^{9})\ln\left(\frac{9.07z(t)}{y(t)} + 1\right)$$

$$t = (8.26 \times 10^{3})\ln\left(r_0 \frac{k}{y(t)}\right) \qquad r_0 = \frac{1}{6.463 \infty 10^{11}}$$

$$\frac{y'}{y} = k - cy \qquad a = ck^{-1} \qquad M = \frac{1}{a} \qquad M = kc^{-1} \qquad \frac{y'}{y} = k - \frac{k}{M}\ y = k(1 - \frac{y}{M})$$

$$y(t) = \frac{Me^{kt}}{C + e^{kt}} \quad \text{or} \quad y(t) = \frac{M}{1 + Ce^{-kt}} \quad \text{where } C = \frac{M}{y_0} - 1$$